Idealised forecast-assimilation experiments and their relevance for convective-scale NWP

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NWP and DA: from large- to convective-scale to ‘idealised’

DA techniques need to evolve in order to keep up with the developments in high-resolution NWP

- increasing resolution is not a panacea: ‘grey zone’ presents many problems...
- more (nonlinear) dynamical processes such as convection, cloud formation, and small-scale gravity waves, are resolved explicitly/partially
- breakdown of dynamical balances (e.g., hydrostatic and geostrophic) at smaller scales
- ensemble-based methods: flow-dependent errors

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It may be unfeasible, and indeed undesirable, to investigate the potential of DA schemes on state-of-the-art NWP models. Instead idealised models can be employed that:

- capture some fundamental processes
- are computationally inexpensive to implement

E.g., 'Idealised' models: hierarchy of complexity

- Lorenz (L63, L95, L2005, ...)
- **SW**/BV/QG models
- simplified NWP models

Using idealised models: approach

1. introduce a physically plausible idealised model and implement numerically
   - Kent et al. (2017): based on the rotating shallow water equations (SWEs) and extending the model of Würsch and Craig (2014) for simplified cumulus cloud dynamics
   - investigate dynamics of the modified model and compare to those of the classical shallow water theory

2. ensemble-based DA - relevant for convective-scale NWP?
   - algorithm: (deterministic) EnKF with techniques to combat sampling errors
   - for relevant experiments:
     - dynamics: set-up, time- and length-scales, ...
     - assimilation: tuning the observing system and ensemble configuration in search of a 'well-tuned' experiment.
     - diagnostics: error-spread statistics, CRPS, observational influence, error-growth statistics...

Houtekamer and Zhang (2016): "*The frontier of data assimilation is at the high spatial and temporal resolution, where we have rapidly developing precipitating systems with complex dynamics*".

Model: summary [*Kent et al., 2017: Tellus A*]

Dynamics:

- when the fluid exceeds threshold heights, dynamics are altered to include a representation of conditional instability (leading to a convective updraft) and idealised moisture transport with associated downdraft and precipitation
- the disruption of large-scale balance principles: (i) breakdown of geostrophic balance in the presence of convection and precipitation; and (ii) breakdown of hydrostatic balance is implicitly enforced by the modified pressure
- initiation of daughter cells away from the parent cell by gravity wave propagation, and convection downstream from an orographic ridge.

Numerics:

- novel, robust and efficient scheme for numerical integration of the model, based on the discontinuous Galerkin finite element method (DGFEM) including methods to ensure well-balancedness and non-negativity of h and r.
- discretises the flow domain into N_{el} elements (defining the horizontal resolution of the model) and uses a dynamic time-step that guarantees stability while allowing for gains in efficiency (i.e., a larger time step) when possible.

Ensemble-based DA for idealised models

GOAL: show that the idealised fluid model provides an interesting testbed for investigating DA algorithms in the presence of convection and precipitation.

I.e., demonstrate a well-tuned idealised ensemble forecast-assimilation system using the model and elucidate its relevance for convective-scale NWP

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State-of-the-art ensemble-based convective-scale NWP and DA: see, e.g., Gustaffson et al. (2018).
What are we aiming for?

Aspect	Operational system
Forecast resolution	$\mathcal{O}(1\,\mathrm{km})$
Update frequency	$\mathcal{O}(1\,\mathrm{hr})$
Ensemble size, N	$\mathcal{O}(10-100)$
# of observations, p	$\mathcal{O}(10^7)$
State dimension, n	$\mathcal{O}(10^9)$
Rank-deficiency	$N \ll p \ll n$
Observation operator	Nonlinear
Localization (horiz.)	$\mathcal{O}(10-100\,\mathrm{km})$
Inflation I	Adaptive/Multipl.
Inflation II	Additive

Diagnostics: Well-tuned?

Tuning (important!):

- operational systems require a great deal of tuning in order to perform optimally
- consider the whole system: forecast model, the observing system, and the assimilation algorithm
- process of developing and arriving at a 'well–tuned' system deserves attention in an idealised setting – **what can we learn?**
- process: iterative adjustments to the observing system and ensemble configuration while monitoring measures for both system performance and, in the case of idealised experiments, relevance.

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1. RMSE of the ensemble mean vs. ensemble spread (SPR): a well-configured ensemble (i.e., sufficiently spread) is crucial to providing an adequate estimation of forecast error. GOAL: $SPR/RMSE \sim 1$.
2. continuous ranked probability score (CRPS): a summary performance measure for probabilistic (ensemble) forecasts. GOAL: low CRPS values.
3. observational influence diagnostic (OID, after Cardinali et al., 2004): sensitivity of the analysis to observations ($\sim 18\%$ in global NWP; $20 - 40\%$ in high-res. NWP). GOAL: $20\% \lessapprox OID \lessapprox 40\%$.
4. error-growth statistics: specifically error-doubling time T_d ($\mathcal{O}(1 \text{ day})$ in global NWP; $\mathcal{O}(1 \text{ hr})$ in high-res. NWP). GOAL: $T_d \sim \mathcal{O}(1 \text{ hr})$.

Idealised experiments: flow over topography

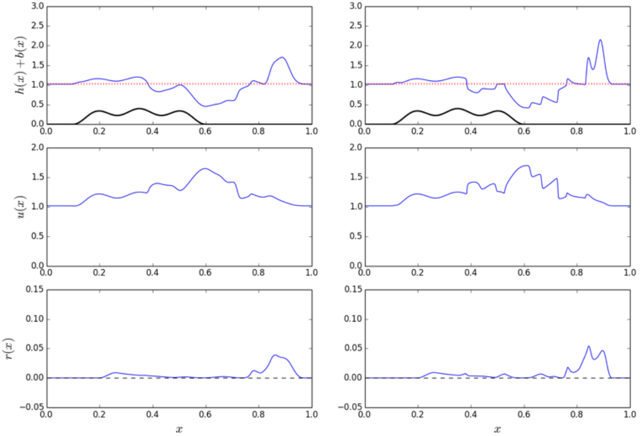
Dynamics: scales and set-up

- non-rotating, supercritical (non-dim. parameters: $\mathrm{Ro} = \infty$, $\mathrm{Fr} = 1.1$)
- length of domain ~ 500 km: then $N_{el} = 200$ implies forecast resolution of ~ 2.5km and $n = \dim(\mathbf{x}) = 600$.
- topography: superposition of sinusoidal waves in part of the domain
- ICS: flat ($h + b = 1$) with 'westerly' wind ($hu = 1$) and no rain $hr = 0$. BCs: periodic.

Assimilation: twin experiments with (deterministic) EnKF (Sakov and Oke, 2008)

- imperfect model scenario: 'nature' run is generated at a finer resolution than the forecast model: $N_{el}^{nat} = \{400, 800\}$
- cycled DA for 48hrs with update frequency of {15, 30, 60} minutes
- variables are observed directly (hence the observation operator is linear, $\mathcal{H} = \mathbf{H}$) with specified error $\sigma = (\sigma_h, \sigma_u, \sigma_r)$ and spatial density in model-space of 40 grid points (~ 100km on forecast grid); i.e., every variable is observed at five (equally spaced and constant) locations, so that $p = 15$.
- to ensure rank–deficiency ($N < p < n$), employ an ensemble with 10 members.
- Filter configuration: making it work!
  - Covariance localisation: length-scale (cf. Gaspari-Cohn)?
  - Covariance inflation: adaptive (RTPS, RTTP; for sampling error) and additive (diagonal $\mathbf{Q}$; for model error)?

An idealised forecast–assimilation system: imperfect model scenario.



Variables h (top), u (middle), and r (bottom) from the forecast model (left, with $N_{el} = 200$) and the nature run (right, with $N_{el} = 800$). The forecast trajectory is smoother and exhibits 'partially-resolved' convection and precipitation while the nature run has sharper 'resolved' features and is a proxy for the truth.

Combinations of the tuning parameters define a single experiment, totalling 2880:

Nature N_{el}^{nat}	{400, 800}
Update frequency [mins]	{15, 30, 60}
Localisation scale, L_{loc}	{0.01, 0.5, 1, 1.5, 2, 3, 4, 6}
Inflation: RTPS	{ 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0}
Additive inflation γ_a	{0.01, 0.05, 0.1, 0.25, 0.5, 1}
Total # of experiments	**2880**

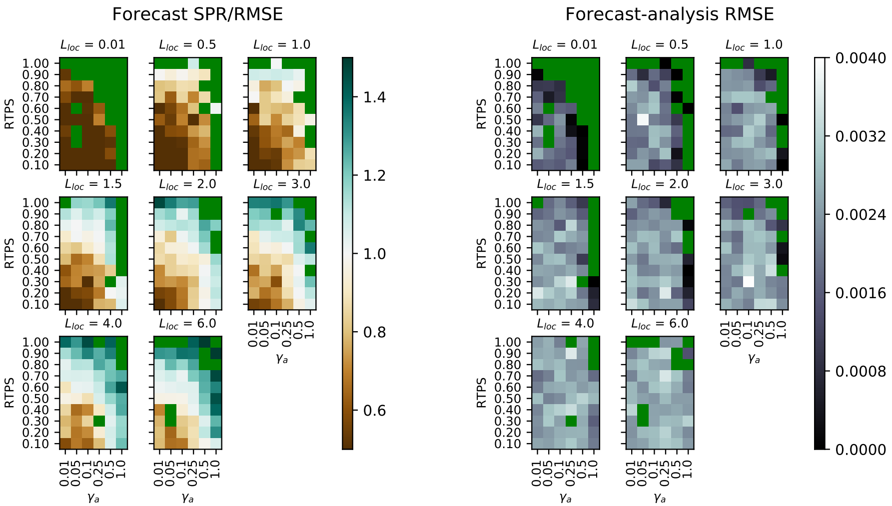
Observing system: defined by the update frequency and resolution of the nature run (as well as obs. error and density)

- 6 observing systems with 480 experiments in each
- experiments in each observing system assimilate the same observations for consistency

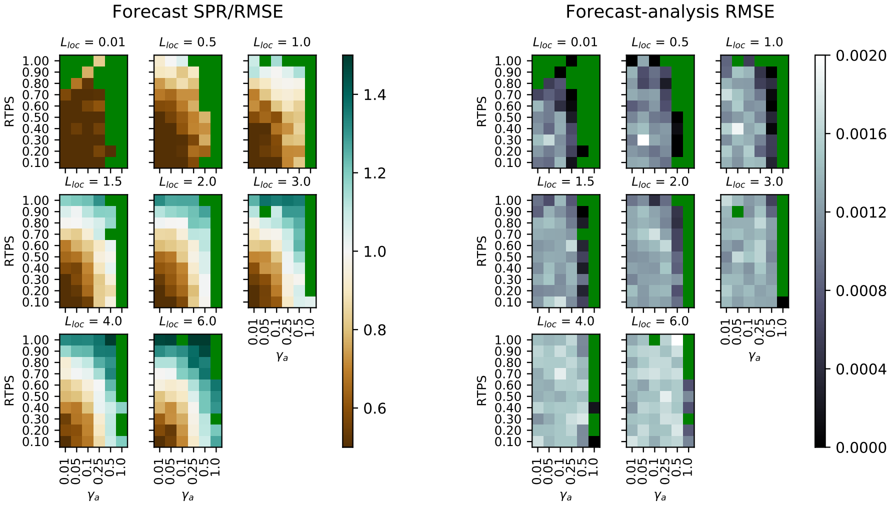
Filter configuration: defined by the localisation and inflation factors.

Number of experiments (out of 480) in which the filter does not diverge for a given observing system:

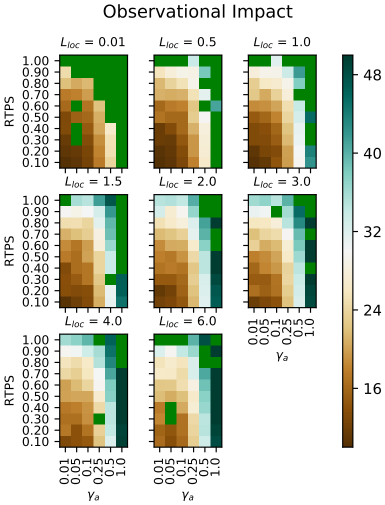
	15 min	30 min	60 min	Total
2:1	381	387	4	772
4:1	121	109	1	231
Total	502	496	5	**1003**

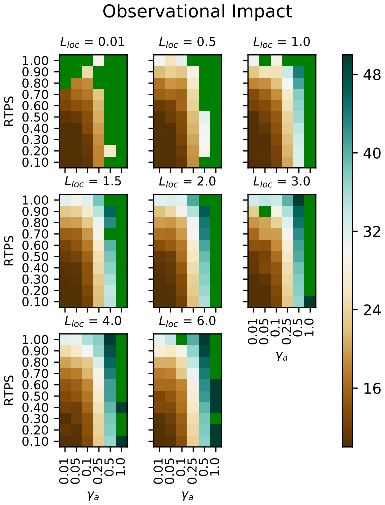


Experiments with the [2:1 – 30min] observing system: spread-error statistics for the forecast (left) and the difference in RMS error of the forecast and analysis (right).



Experiments with the [2:1 – 15min] observing system: spread-error statistics for the forecast (left) and the difference in RMS error of the forecast and analysis (right).



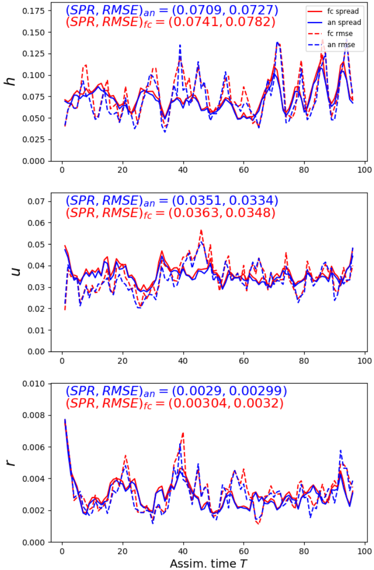


Experiments with the [2:1 – 30min] (left) and [2:1 – 15min] (right) observing system: observational influence diagnostic. Recall: target is $20\% \lessapprox OID \lessapprox 40\%$.

loc_6.0_add_0.1_rtpp_0.5_rtps_0.5

Domain-averaged RMS error (dashed) and spread (solid) time series for the forecast (red) and analysis (blue) ensemble: h (top), u (middle), and r (bottom).

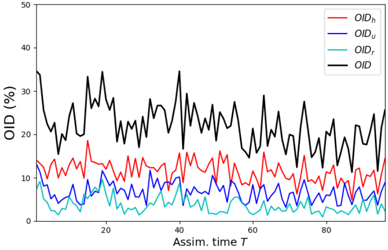
- each series comprises 96 cycles (i.e., 48 hours updated every 30mins);
- time-averaged values are given in the top-left corner of each panel;
- the ensemble spread is similar in magnitude to the RMSE of the ensemble mean for both the forecast (red) and analysis (blue);
- the filter produces an analysis ensemble with lower error than the forecast ensemble for all model variables.

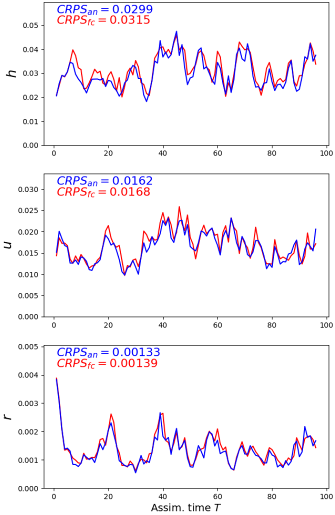


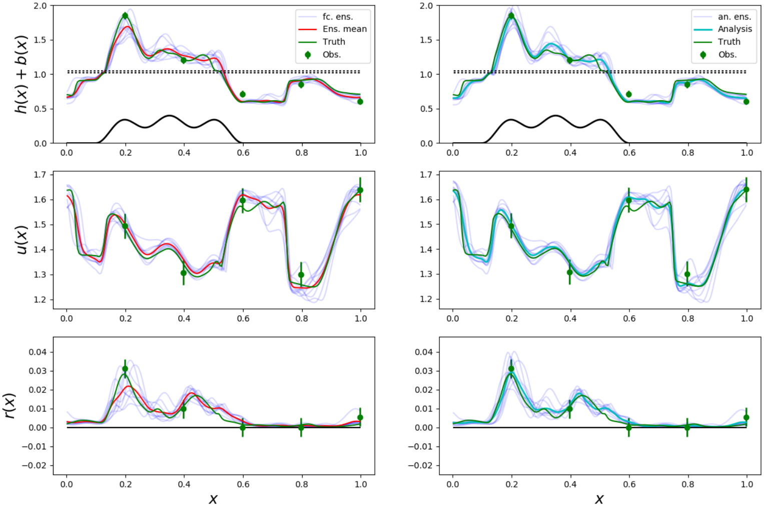
Right: domain-averaged CRPS time series for the forecast (red) and analysis (blue) ensemble: h (top), u (middle), and r (bottom).

- each series comprises 96 cycles (i.e., 48 hours updated every 30mins); time-averaged values are given in the top-left corner of each panel;
- on average, the assimilation update marginally improves the reliability of the ensemble.

Below: domain-averaged OID time series (overall and for each variable)

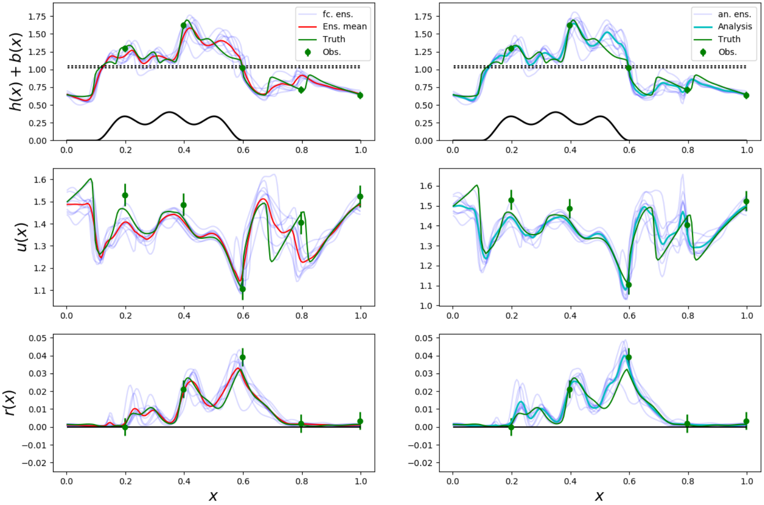






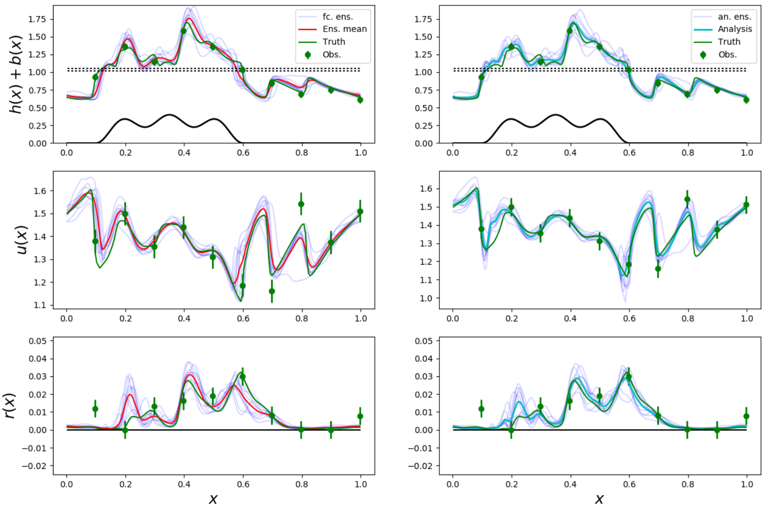
Ensemble trajectories (blue) and their mean (red for forecast; cyan for analysis), pseudo-observations (green circles with corresponding error bars), and nature run (green solid line) after 65 cycles. Left: forecast. Right: analysis.

loc_6.0_add_0.1_rtpp_0.5_rtps_0.5



Ensemble trajectories (blue) and mean (red forecast; cyan analysis), pseudo-obs. (green circles with error bars), and nature run (green solid line) after 51 cycles. Left: forecast. Right: analysis. **Observe every 40 gridpoints:** ~ 100km.

`loc_2.0_add_0.25_rtpp_0.5_rtps_0.6`



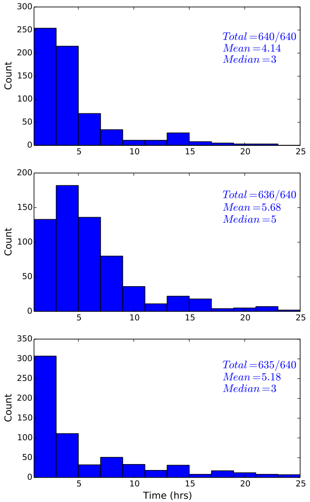
Ensemble trajectories (blue) and mean (red forecast; cyan analysis), pseudo-obs. (green circles with error bars), and nature run (green solid line) after 51 cycles. Left: forecast. Right: analysis. **Observe every 20 gridpoints:** ~ 50km.

`loc_2.0_add_0.25_rtpp_0.5_rtps_0.6`

Error-doubling time T_d

- moist convection severely limits mesoscale predictability: for limited–area cloud–resolving models: mean error–doubling time ~ 4 hours.
- ensemble forecasts initialised with the analysis perturbations from a well-tuned experiment should exhibit characteristic error growth rates on this timescale
- compute T_d for an idealised ensemble prediction system by running numerous staggered forecasts initialised with the analysis increments

NOTE: different set-up



'Protocol' revisited and results

Kent et al., 2017: idealised model simulates some fundamental dynamical processes associated with convecting and precipitating weather systems, suggesting that it is a suitable candidate for investigating DA algorithms at convective scales.

Aspect	Operational system	Our idealised system	Relevant?
Forecast resolution	$\mathcal{O}(1\text{ km})$	2.5 km	✓
Update frequency	$\mathcal{O}(1\text{ hr})$	30 mins	✓
Ensemble size, N	$\mathcal{O}(10-100)$	10	–
# of observations, p	$\mathcal{O}(10^7)$	15	N/A
State dimension, n	$\mathcal{O}(10^9)$	600	N/A
Rank-deficiency	$N \ll p \ll n$	$N < p < n$	✓
Observation operator	Nonlinear	Linear	✗
Localization	$\mathcal{O}(10-100\text{ km})$	$\sim 100\text{ km}$	✓
Inflation I	Adaptive	RTPS $= 0.4 - 0.7$	–
Inflation II	Additive	$\gamma_a = 0.1 - 0.5$	–
$SPR/RMSE$	~ 1	~ 1	✓
Observational influence	$20\% \lessapprox OID \lessapprox 40\%$	$\sim 25\%$	✓
Error-doubling time, T_d	$\mathcal{O}(1\text{ hr})$	$\sim 4\text{ hrs}$	✓

This study: advocates this further by conducting numerous forecast-assimilation experiments, providing a critical assessment of their performance, and addressing their relevance for convective-scale NWP.

Conclusion + ideas

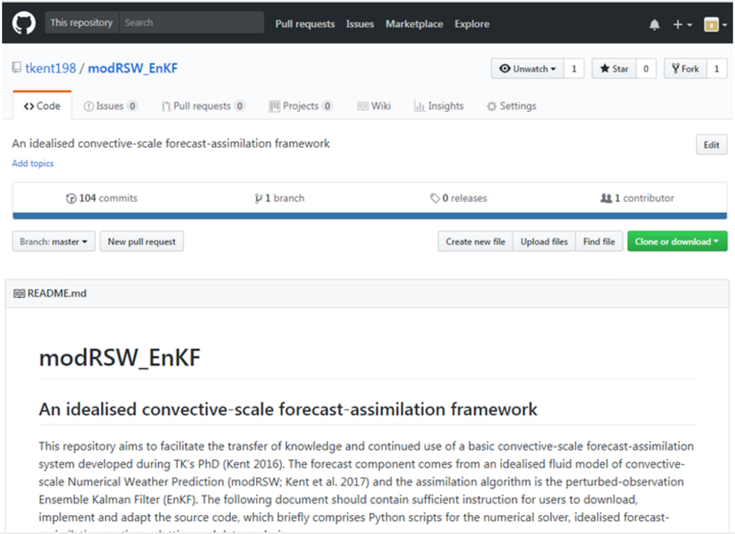
Summary:

- the idealised fluid model provides an interesting testbed for forecast-assimilation experiments with characteristics relevant for convective-scale NWP
- tuning: for performance and relevance
- filter performs adequately (given the relatively simple set-up and strong nonlinearities of the model): certain configurations yield ensembles that estimate the forecast errors well
- additive and adaptive inflation crucial for satisfactory filter performance; localisation also crucial due to rank-deficiency (limited ensemble size)
- observational influence can be tuned to be similar to that of operational NWP and error-growth rates comparable to convective-scale NWP systems
- myriad set-ups to investigate: **valuable learning process in a clean environment!**

Ideas / future work:

- exploring satellite DA (**Cantarello et al., X4.291 this session 14:00-15:45**)
- investigating 'representation' errors
- nonlinear DA: comparison of methods etc.
- import into DA frameworks...?

GitHub repository: modRSW_EnKF



Thanks very much for your attention ... questions?

References:

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- Kent (2017): **'An idealised convective-scale forecast-assimilation framework'**, `https://github.com/tkent198/modRSW_EnKF`
- Gustafsson et al. (2018): Survey of data assimilation methods for convective-scale NWP at operational centres. *QJRMS*, 144(713), 1218-1256.
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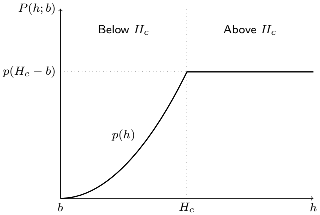
- Würsch and Craig (2014): A simple dynamical model of cumulus convection for data assimilation research. *Meteorologische Zeitschrift*, 23(5):483-490.

Modified SWEs

- two threshold heights $H_c < H_r$: when fluid exceeds these heights, different mechanisms kick in and alter the classical SW dynamics
- modifications to the effective pressure gradient (via SW pressure: $p(h) = \frac{1}{2}gh^2$) in the momentum equation
- extra equation for the conservation of model 'rain' to close the system



Modified pressure $P(h;b)$: $p(H_c - b) = \frac{1}{2}g(H_c - b)^2$ above the threshold H_c is lower than the standard pressure $p(h) = \frac{1}{2}gh^2$, thus forcing the fluid to rise where $h + b > H_c$.

$$\partial_t h + \partial_x(hu) = 0,$$

$$\partial_t(hu) + \partial_x(hu^2 + P) + hc_0^2\partial_x r - fhv = -Q\partial_x b,$$

$$\partial_t(hv) + \partial_x(huv) + fhu = 0,$$

$$\partial_t(hr) + \partial_x(hur) + h\widetilde{\beta}\partial_x u + \alpha hr = 0,$$

where $P = P(h;b) = \begin{cases} p(H_c - b), & \text{for } h + b > H_c, \\ p(h), & \text{otherwise,} \end{cases}$

$$Q = Q(h;b) = \begin{cases} p'(H_c - b), & \text{for } h + b > H_c, \\ p'(h), & \text{otherwise,} \end{cases}$$

and $\widetilde{\beta} = \begin{cases} \beta, & \text{for } h + b > H_r,\ \partial_x u < 0, \\ 0, & \text{otherwise.} \end{cases}$

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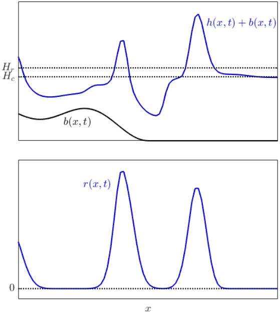

Schematic solutions: convection (in h field) and associated 'rain'

$$\partial_t h + \partial_x(hu) = 0,$$
$$\partial_t(hu) + \partial_x(hu^2 + P) + hc_0^2\partial_x r - fhv = -Q\partial_x b,$$
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Non-dimensionalised modRSW equations

$$
\begin{aligned}
&\partial_t h + \partial_x (hu) = 0, \\
&\partial_t (hu) + \partial_x (hu^2 + P) + Q\partial_x b + h\widetilde{c_0}^2 \partial_x r - \frac{1}{\mathrm{Ro}} hv = 0, \\
&\partial_t (hv) + \partial_x (huv) + \frac{1}{\mathrm{Ro}} hu = 0, \\
&\partial_t (hr) + \partial_x (hur) + h\widetilde{\beta}\partial_x u + \widetilde{\alpha} hr = 0,
\end{aligned}
$$

where:

$$
\begin{aligned}
P(h,b) &= \frac{1}{2\mathrm{Fr}^2}\left[h^2 + ((H_c - b)^2 - h^2)\Theta(h + b - H_c)\right], \\
Q(h,b) &= \frac{1}{\mathrm{Fr}^2}\left[h + (H_c - b - h)\Theta(h + b - H_c)\right], \\
\widetilde{\beta} &= \beta\Theta(h + b - H_r)\Theta(-\partial_x u).
\end{aligned}
$$

$\Theta(x) = 1$ if $x > 0$; and 0 if $x \leq 0$, and the following parameters are introduced:

$$
\mathrm{Fr} = \frac{V_0}{\sqrt{gH_0}}, \qquad \mathrm{Ro} = \frac{V_0}{fL_0}, \qquad \widetilde{c_0}^2 = \frac{c_0^2}{V_0^2}, \qquad \widetilde{\alpha} = \frac{L_0}{V_0}\alpha.
$$

Some theoretical aspects

Eigenanalysis: shallow water systems are hyperbolic, and can thus be solved via a range of numerical recipes for hyperbolic systems. What about the modified system?

$$\partial_t \boldsymbol{U} + \partial_x \boldsymbol{F}(\boldsymbol{U}) + \boldsymbol{G}(\boldsymbol{U})\partial_x \boldsymbol{U} + \boldsymbol{S}(\boldsymbol{U}) = 0,$$

Hyperbolicity determined by eigen-structure: (all eigenvalues must be real).
Eigenvalues of the system are determined by the Jacobian matrix $\partial \boldsymbol{F}/\partial \boldsymbol{U} + \boldsymbol{G}(\boldsymbol{U})$:

$$\lambda_{1,2} = u \pm \sqrt{\partial_h P + c_0^2 \widetilde{\beta}}, \quad \lambda_{3,4} = u.$$

Since $P(h;b)$ is non-decreasing and $\widetilde{\beta}$ non-negative, the eigenvalues are real. Hence, the modified SW model is (weakly) hyperbolic.

Wave speeds: waves travelling through (saturated) regions of convection slow down

- $h + b < H_c$: $\partial_h P = gh$, $\widetilde{\beta} = 0$ implies standard eigenvalues $\lambda_{1,2} = u \pm \sqrt{gh}$
- $H_c < h + b < H_r$: $\partial_h P = 0$, $\widetilde{\beta} = 0$ implies modified eigenvalues $\lambda_{1,2} = u$
- $H_r < h + b$: $\partial_h P = 0$ and $\widetilde{\beta} = \beta$ implies modified eigenvalues $\lambda_{1,2} = u \pm \sqrt{c_0^2 \beta}$

Scaling

For a given Fr potential **characteristic scales of the dynamics** can be analysed and, where possible, likened to high-resolution NWP.

- Consider a fixed length of domain $L_0 = 500$ km and velocity-scale $V_0 \sim 20$ ms^{-1}, implying a time-scale $T_0 \sim 25000$ seconds ($\sim 6.94...$ hours). Thus, one hour is equal to 0.144 non-dimensional time units.
- $\mathrm{Fr} = 1.1$ implies $gH_0 \sim 330$ m^{2}s^{-2}. We note that gravity g is actually a reduced gravity g', and impose a height scale of $H_0 \sim 500\,\mathrm{m}$.
- This implies a reduced gravity $g' \sim 0.66\,\mathrm{ms}^{-1}$ and is justified as follows. Taking the average air density in the layers 0-500 and 500-1500 meters from the air density profile of the International Standard Atmosphere*, $\rho_{0-500m} = 1.196\,\mathrm{kg\,m}^{-3}$ and $\rho_{500-1500m} = 1.113\,\mathrm{kg\,m}^{-3}$, gives a reduced gravity of $g' \sim 0.68\,\mathrm{ms}^{-1}$.
- The threshold height H_c mimics the Level of Free Convection (LFC), which can be on the order of several hundred meters in convectively unstable conditions.

*https://onlinelibrary.wiley.com/doi/pdf/10.1002/9781118568101.app2

Two popular flavours of **adaptive inflation**: 'Relaxation To Prior Perturbation' (RTTP) and 'Relaxation To Prior Spread' (RTPS) methods.

RTTP: relaxes analysis perturbations $\mathbf{X}_j^a$ back to the forecast perturbations $\mathbf{X}_j^f$ independently at each analysis point:

$$\mathbf{X}_j^a \leftarrow (1-\alpha)\mathbf{X}_j^a + \alpha\mathbf{X}_j^f, \tag{2}$$

where $\alpha \in [0, 1]$ is a tuneable parameter.This amounts to a combination of multiplicative and additive inflation.

RTPS: relaxes the analysis ensemble spread back to the forecast spread:

$$\sigma^a \leftarrow (1-\alpha)\sigma^a + \alpha\sigma^f, \tag{3}$$

where σ is the spread at each gridpoint (i.e., standard deviation)and $\alpha \in [0, 1]$ is a tuneable parameter. This is a purely multiplicative form of inflation.

NOTE: the DEnKF is equivalent to the 'no-perturbation' EnKF with implicit RTPP adaptive inflation with $\alpha = 1/2$

Additive inflation comprises adding random Gaussian perturbations $\eta_j \sim \mathcal{N}(0, \gamma_a \mathbf{Q})$ during, or at the end of, the forecast step:

$$\mathbf{x}_j(t_i) = \mathcal{M}(\mathbf{x}_j(t_{i-1})) + \eta_j, \quad j = 1, ..., N, \tag{4}$$

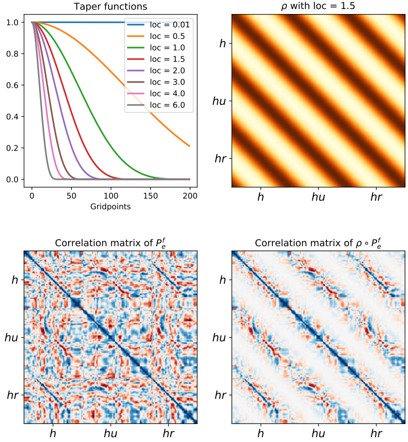
where the forecast–model error matrix $\mathbf{Q}$ is prescribed from some knowledge of the modeling system and γ_a is a tuneable parameter controlling the overall magnitude of the sample perturbations. How one best defines $\mathbf{Q}$ is an open question; often a static matrix developed offline from historical analysis increments. In this study, $\mathbf{Q}$ is diagonal and contains the square of the (imposed) observational error of each variable:

$$\mathbf{Q} = \begin{pmatrix} \boldsymbol{\sigma}_h^2 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\sigma}_u^2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\sigma}_r^2 \end{pmatrix}, \tag{5}$$

where $\boldsymbol{\sigma}_h^2$, $\boldsymbol{\sigma}_u^2$ and $\boldsymbol{\sigma}_r^2$ are diagonal matrices of size $N_{el} \times N_{el}$. In order not to introduce bias into the model state $\mathbf{x}(t_i)$, an unbiased additive inflation $\tilde{\eta}_j$ is computed and used in (**??**) by subtracting the ensemble averaged additive inflation $\overline{\eta}$:

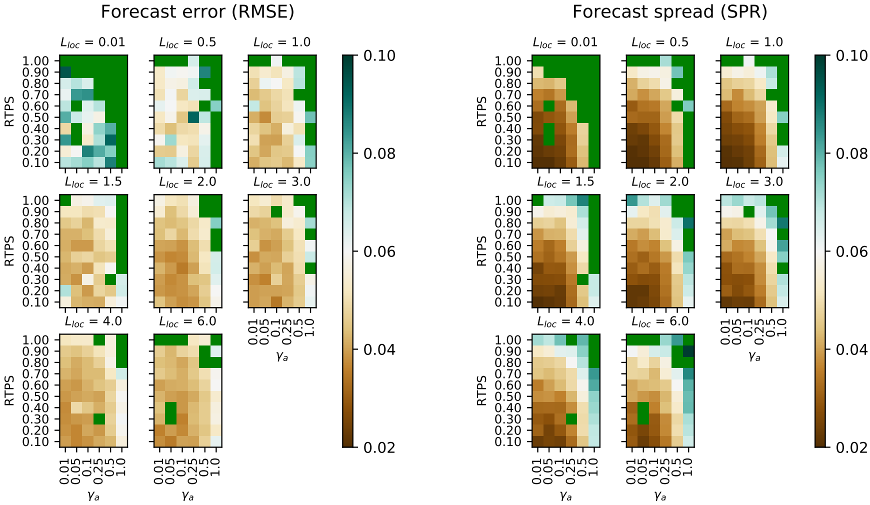
$$\tilde{\eta}_j = \eta_j - \overline{\eta}. \tag{6}$$

Additive inflation does not try to represent the model error explicitly, but acts in some sense as a lower bound for the forecast error, thus preventing filter divergence.

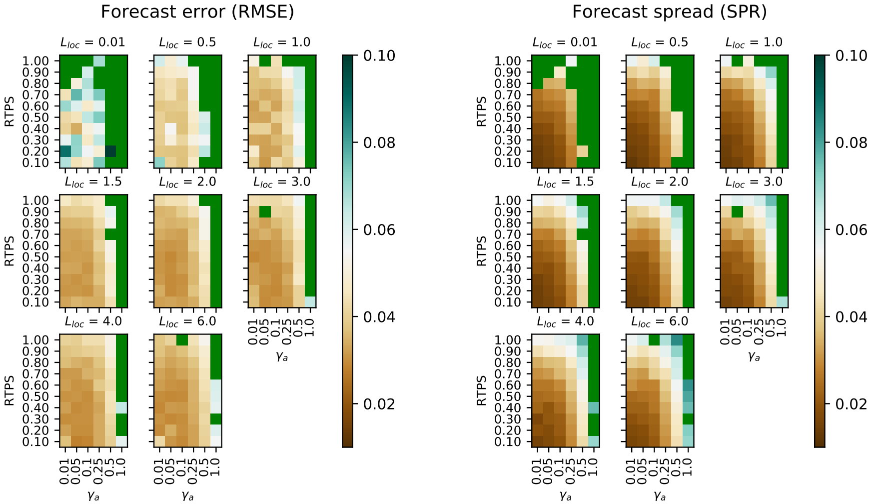


Ingredients for covariance localization.

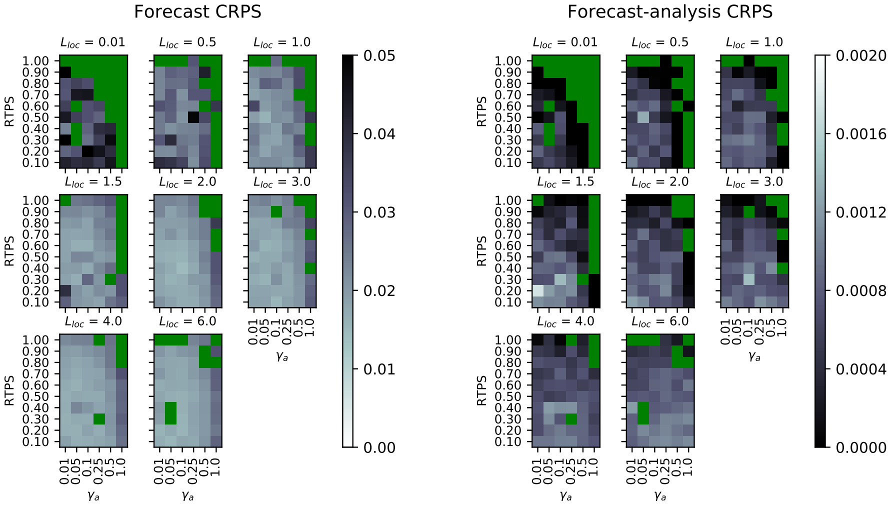
Top left: Gaspari-Cohn taper functions $\rho = \rho(L_{loc})$ for various L_{loc} values, which define a lengthscale in model space (x-axis). No localization (blue) corresponds to $L_{loc} = 0.01$; $L_{loc} = 1$ (green) defines a lengthscale of $N_{el}/L_{loc} = 200$ grid points. Increasing L_{loc} leads to a more severe taper and reduction to zero. Top right: banded localization matrix $\boldsymbol{\rho}$ defined by $L_{loc} = 1.5$, which has the same dimension as $\mathbf{P}_e^f$ and is a block 3×3 matrix for our 3-variable system. Bottom left: a time-dependent correlation matrix (i.e., normalized $\mathbf{P}_e^f$) generated by a 10-member ensemble. Bottom right: the localized correlation matrix (i.e., normalized $\mathbf{P}_{loc}^f = \boldsymbol{\rho} \circ \mathbf{P}_e^f$) combines the top right and bottom left matrices. Long range correlations deemed spurious are set to zero beyond a certain distance in model space.



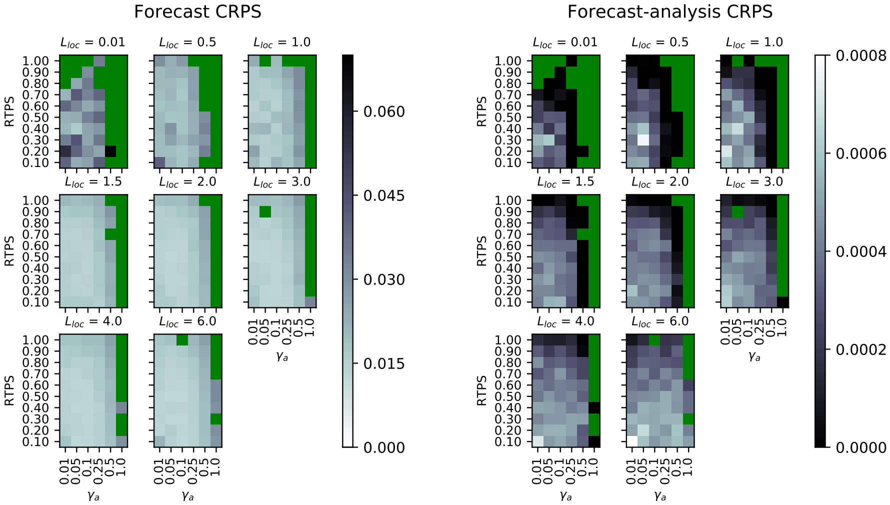
Experiments with the [2:1 – 30min] observing system: RMSE error of the forecast ensemble mean (left) and the spread of the forecast ensemble (right).



Experiments with the [2:1 – 15min] observing system: RMSE error of the forecast ensemble mean (left) and the spread of the forecast ensemble (right).



Experiments with the [2:1 – 30min] observing system: CRPS for the forecast ensemble (left) and the difference in CRPS of the forecast and analysis (right).



Experiments with the [2:1 – 15min] observing system: CRPS for the forecast ensemble (left) and the difference in CRPS of the forecast and analysis (right).

Numerics: methodology

- Rhebergen et al. (2008) developed a novel discontinuous Galerkin (DG) finite element framework for non-conservative hyperbolic system of PDEs, deals robustly with high nonlinearity and non-conservative products, $\boldsymbol{G}(\boldsymbol{U})\partial_x\boldsymbol{U}$
- we combine this with the scheme of Audusse et al. (2004) to discretise topography that maintains well-balancedness and preserves non-negativity of fluid depth and rain variable