A modified shallow water system for investigating convective-scale data assimilation

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1. Background

Data assimilation (DA) techniques need to evolve in order to keep up with the developments in **convection-permitting Numerical Weather Prediction** (NWP) models. It is often unfeasible, and indeed undesirable, to investigate the potential of new data assimilation schemes on operational forecasting systems. Instead, **idealised models** can be employed that capture the fundamental features of **convective-scale dynamics** while remaining computationally inexpensive, thus allowing an extensive investigation of the proposed scheme.

3. Model: SWEs with 'rain'

We outline an idealised NWP model (after [1]) based on the SWEs which is designed to represent an **idealised atmosphere with moist convection**. Moisture is incorporated into the traditional SWEs by the introduction of a **'rain mass fraction'** variable, r, which acts on the momentum equation (1b) via the geopotential, and is allowed to precipitate via an additional conservation equation (1c).

$$\partial_t h + \partial_x(hu) = 0, \quad (1a)$$

$$\partial_t(hu) + \partial_x(hu^2 + p(h)) + gh\partial_x b + hc_0^2\partial_x r - fhv = 0, \quad (1b)$$

$$\partial_t(hr) + \partial_x(hur) + h\widetilde{\beta}\partial_x u + \alpha hr = 0, \quad (1c)$$

$$\partial_t(hv) + \partial_x(huv) + fhu = 0, \quad (1d)$$

$$\partial_t b = 0, \quad (1e)$$

where $p = p(h)$ is a pseudo-pressure defined by:

$$p(h) = \begin{cases} \frac{1}{2}gH_c^2, & \text{for } h + b > H_c, \\ \frac{1}{2}gh^2, & \text{otherwise,} \end{cases}$$

and:

$$\widetilde{\beta}(h, \partial_x u) = \begin{cases} \beta, & \text{for } h + b > H_r \text{ and } \partial_x u < 0, \\ 0, & \text{otherwise.} \end{cases}$$

- h = fluid depth, (u, v) = velocities, b = bottom topography, r = rain mass fraction; all as a function of (x, t).
- H_c, H_r = threshold heights, above which convection and 'rain' processes occur.
- $c_0^2 = gH_r$, α = rate of 'rain' removal (by precipitation), β controls 'rain' formation, f = Coriolis frequency, g = gravity.

4. Why SWEs?

The **traditional** SWEs are:

- often used in the meteorological community for modelling atmospheric circulation.
- fully nonlinear and sufficiently **chaotic** for meaningful DA.
- admit **gravity waves** and have **scale interactions** (fast/slow modes).
- admit discontinuous solutions, akin to the **propagation of atmospheric fronts**.

The **modified** SWEs:

- artificially mimic **conditional instability** (positive buoyancy) and the transport of **moisture**.
- contain **switches** for the onset of convection and precipitation - realistic features of operational NWP models.

2. Approach

1. Introduce a **physically plausible** idealised model and implement numerically.
   - based on the shallow water equations (SWEs).
   - produce **'synthetic observations'** from the model variables for use in DA experiments (linear and nonlinear).
2. Construct some DA experiments based on **ensemble (En)** and **variational (VAR)** techniques.
   - integrate model into Met Office's Python/Fortran framework for idealised DA experiments.
   - investigate potential for new control variable transform, e.g. based on scale separation (heterogeneous background error covariance).

5. Theoretical and numerical aspects

The **quasi-2D rotating symmetric system** can be written in a non-conservative formulation:

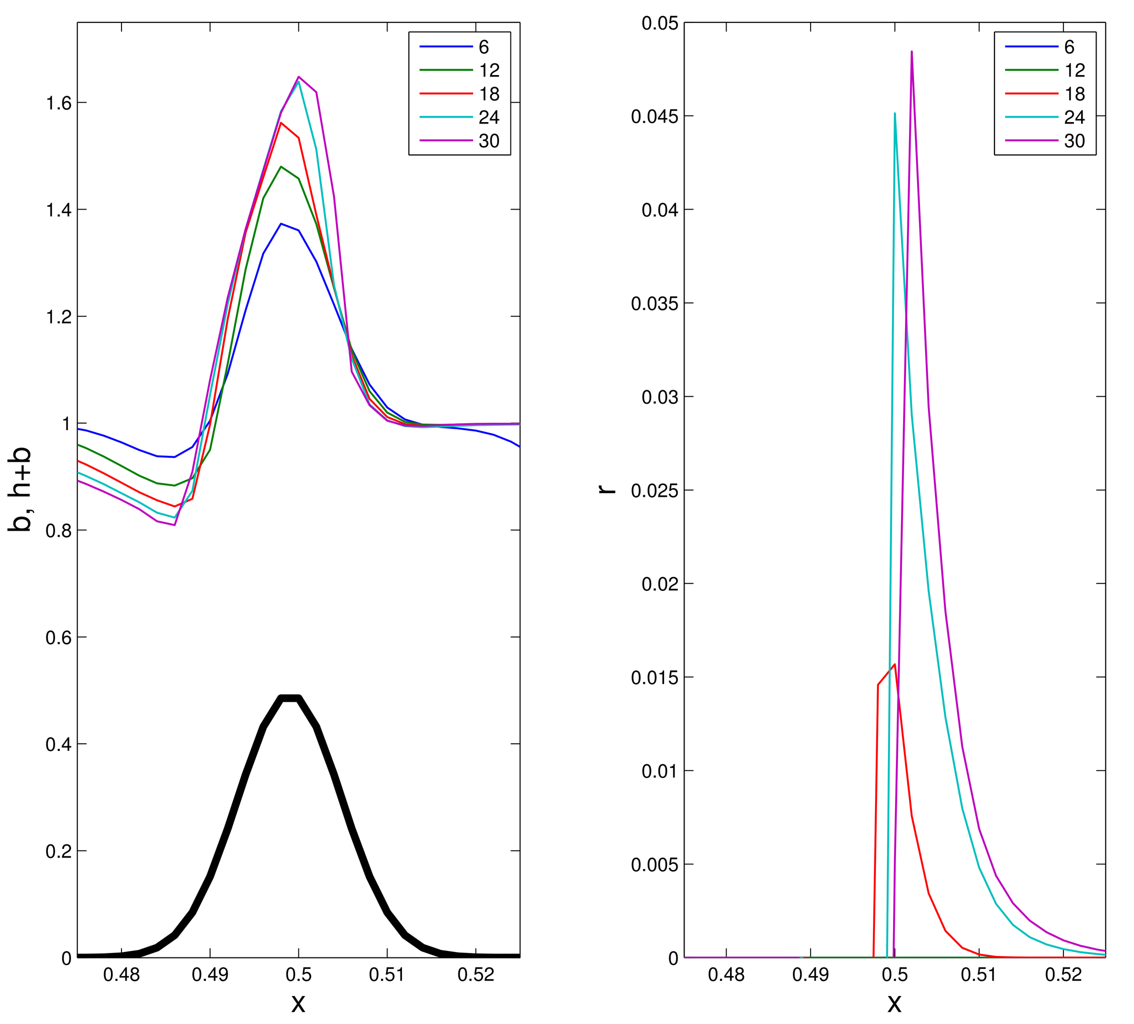
$$\partial_t \mathsf{U} + \partial_x \mathsf{F}(\mathsf{U}) + \mathsf{G}(\mathsf{U})\partial_x \mathsf{U} + \mathsf{S}(\mathsf{U}) = 0$$

with:

$$\mathsf{U} = \begin{bmatrix} h \\ hu \\ hr \\ hv \\ b \end{bmatrix}, \quad \mathsf{F}(\mathsf{U}) = \begin{bmatrix} hu \\ hu^2 + p(h) \\ hur \\ huv \\ 0 \end{bmatrix}, \quad \mathsf{G}(\mathsf{U}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -c_0^2 r & 0 & c_0^2 & 0 & gh \\ -\widetilde{\beta}u & \widetilde{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathsf{S}(\mathsf{U}) = \begin{bmatrix} 0 \\ -fhv \\ \alpha hr \\ fhu \\ 0 \end{bmatrix}.$$

- valid **non-conservative hyperbolic system** of partial differential equations.
- solved numerically using a shock-capturing finite volume/element framework [2].
- deals robustly with high non-linearity and non-conservative products, $\mathsf{G}(\mathsf{U})\partial_x \mathsf{U}$.
- $p(h)$ and $\widetilde{\beta}$ describe **onset of convection (updraft)** and **rain formation** respectively.
- combines **non-linearity due to the switches** and the genuine **hydrodynamic non-linearity** of the SWEs.

6. Numerical results



- Flow over topography (single 'mountain'; thick black line) without rotation.
- Model set-up (non-dimensionalised):
  - characteristic scales: $L = 500km, V = 10m/s, H = 100m$.
  - 500 gridcells, akin to 1km horizontal resolution attained in convective-scale NWP.
  - critical heights: $H_c = 102m, H_r = 105m$.
  - very large Rossby number (no rotation); Froude number = 0.3.
- Initially: height profile is flat, uniform flow from west to east. As fluid exceeds: H_c there is upward forcing, H_r 'rain' can form (given convergence, not shown). Coloured lines correspond to time in minutes.

7. Next steps

- Further numerical tests and analysis of the dynamical properties of the system, including effect of rotation.
- Integrate model into the Met Office's Python/Fortran idealised DA modelling framework (employed in recent DA study [3]).
  - ensemble Kalman Filter.
  - 4D-VAR experiment using quasi-static covariance estimation.
  - control variable transform using scale separation for fast/slow modes.

References

[1] Würsch, M., and Craig, G.C.: A simple dynamical model of cumulus convection for data assimilation research. *Submitted to: Meteorologische Zeitschrift.*

[2] Rhebergen, S., Bokhove, O., and Van der Vegt, J., 2008: Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations. *J. Comp. Phys.*, 227(3), 1887-1922.

[3] Fairbairn, D., Pring, S. R., Lorenc, A. C., and Roulstone, I., 2014: A comparison of 4DVar with ensemble data assimilation methods. *QJRMS*, 140(678), 281-294.

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