Introducing the ‘ABC model’ for the efficient study of convective-scale data assimilation techniques

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Convective-Scale NWP (km grid scale)

Some challenges compared to large-scale NWP

- Dynamics:
  - Highly non-linear, 3D, compressible flow with short predictability timescale.
  - Acoustic modes appear in the solution to model equations (clever solvers needed).
  - Near-hydrostatic balance (non-convecting regions).
  - Non-hydrostatic flow (convective regions).
  - Significant ageostrophic component on the small scale.
  - Small-scale quantities become important (e.g. vertical motion, hydrometeors).
- Important observations:
  - Radar reflectivity, IR/MW sounders/imagers.
  - High-density observations (correlated errors).

The ‘ABC’ Simplified Model

Research brief for simplified model to facilitate convective-scale DA research

- Starting point: the 3D compressible Euler equations.
- Non-hydrostatic, compressible.
- Unbalanced on small scales, balanced on large scales.
- Conserve mass and energy:

$$\frac{d}{dt}\int \rho(x,z,t)dxdz = 0 \qquad \frac{d}{dt}\int E(x,z,t)dxdz = 0.$$

- Cheap to run:
  - Integrated with an explicit numerical scheme.
  - 2D (longitude/height domain).
- Dry dynamics (currently).

ABC Model: Tunable Parameters, A, B, C

$$
\begin{aligned}
\frac{\partial u}{\partial t} + B\mathbf{u}\cdot\nabla u + C\frac{\partial \tilde{\rho}'}{\partial x} - fv &= 0 \\
\frac{\partial v}{\partial t} + B\mathbf{u}\cdot\nabla v + fu &= 0 \\
\frac{\partial w}{\partial t} + B\mathbf{u}\cdot\nabla w + C\frac{\partial \tilde{\rho}'}{\partial z} - b' &= 0 \\
\frac{\partial \tilde{\rho}'}{\partial t} + B\nabla\cdot(\tilde{\rho}\mathbf{u}) &= 0 \\
\frac{\partial b'}{\partial t} + B\mathbf{u}\cdot\nabla b' + A^2 w &= 0 \\
p' = C\rho_0\tilde{\rho}'
\end{aligned}
$$

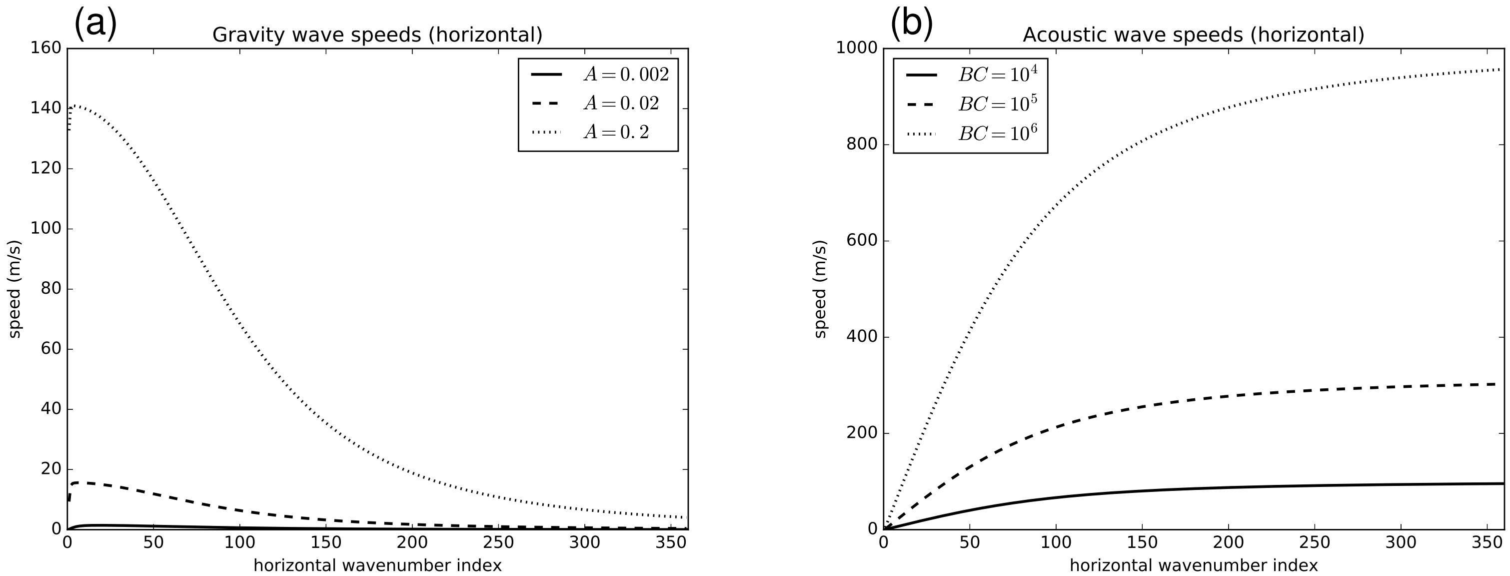
- $\mathbf{u} = (u, v, w)$.
- $\tilde{\rho}'$ scaled density pert: $\tilde{\rho}' = (\rho - \rho_0)/\rho_0$.
- b' buoyancy: $b' = (\theta - [\theta_R + \theta_0(z)])\, g/\theta_R$.
- A pure gravity wave frequency.
- B modulation $0 < B \leq 1$, to reduce acoustic wave frequencies.
- $\sqrt{BC}$ pure acoustic wave speed.

Most experiments: $A = 0.02\,\mathrm{s}^{-1}$, $B = 0.01$, $C = 10000\,\mathrm{m}^2\mathrm{s}^{-2}$.

See Petrie, Bannister, and Cullen (2017).

Model behaviour — linear modes

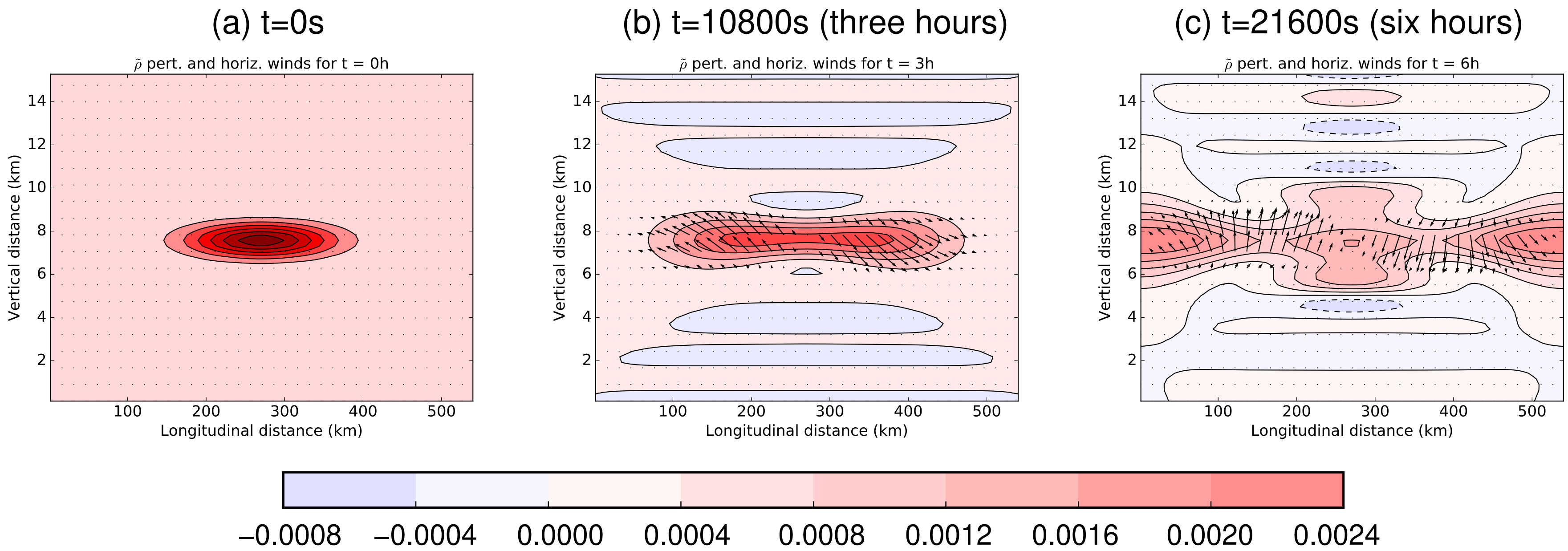
Horizontal gravity group wave speed	Horizontal acoustic group wave speed



Recall: $\sqrt{BC}$ pure acoustic wave speed

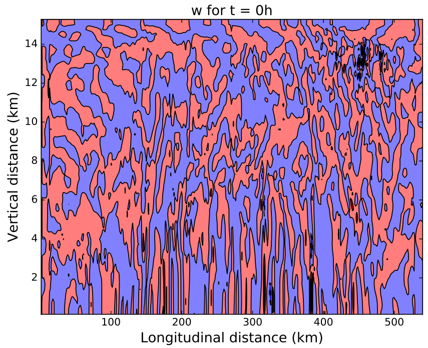
Model behaviour — idealised large-scale

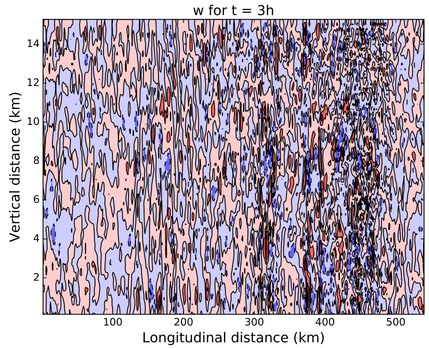
Scaled density pert. $(\tilde{\rho}')$ and horiz. winds (u, v)

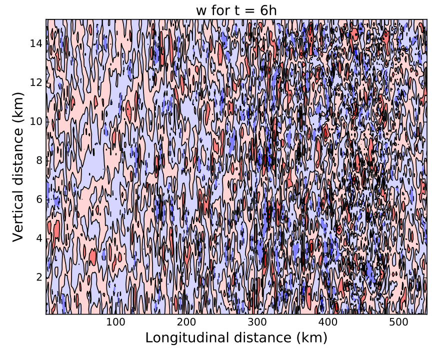


Model behaviour — example integration

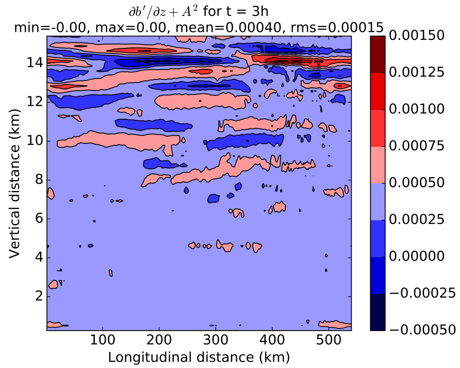
$w\ (t=0)$	$w\ (t=3\ \mathrm{h})$	$w\ (t=6\ \mathrm{h})$





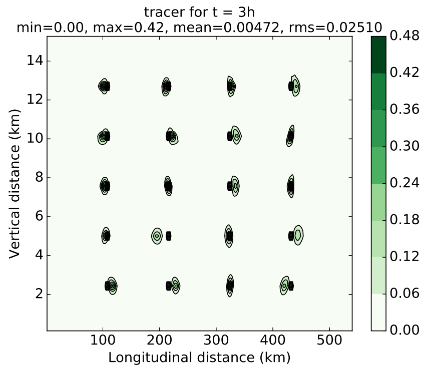


Effective buoyancy



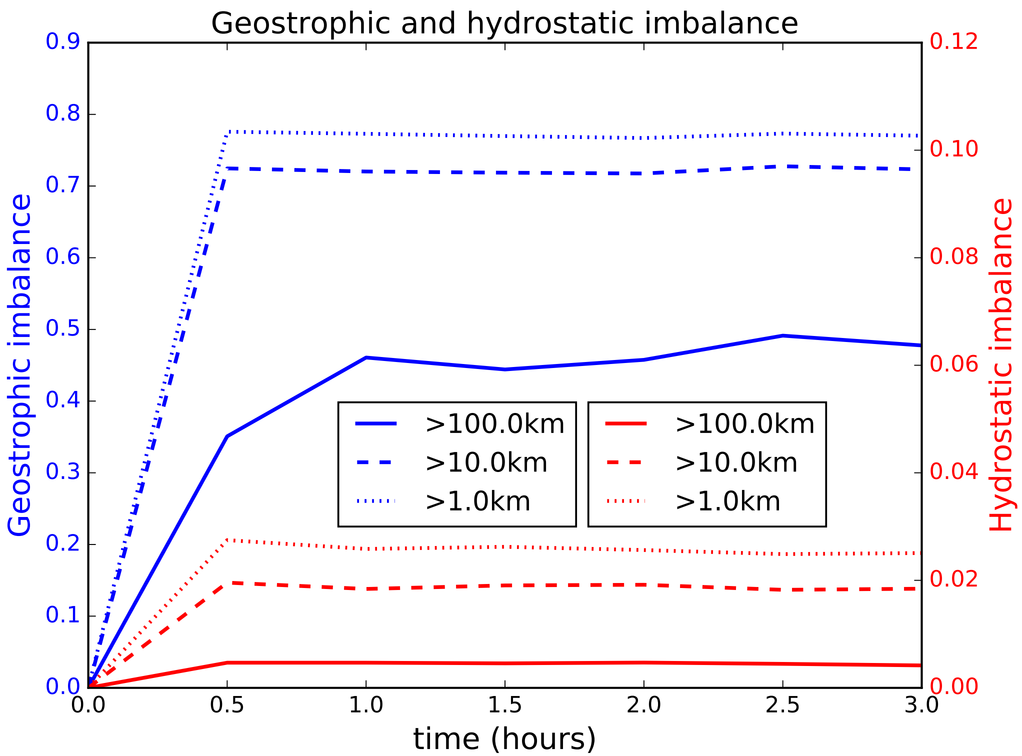
$$\frac{\partial}{\partial z}\left[b_0(z) + b'(x,z,t)\right] = A^2 + \frac{\partial}{\partial z} b'(x,z,t)$$

Tracer



Model behaviour — imbalance

Geostrophic and hydrostatic imbalances



$$\text{geo. imbal.} = \frac{\mathrm{RMS}\left(C\frac{\partial\tilde{\rho}'}{\partial x} - fv\right)}{\mathrm{RMS}\left(C\frac{\partial\tilde{\rho}'}{\partial x}\right) + \mathrm{RMS}\left(fv\right)}$$

$$\text{hydro. imbal.} = \frac{\mathrm{RMS}\left(C\frac{\partial\tilde{\rho}'}{\partial z} - b'\right)}{\mathrm{RMS}\left(C\frac{\partial\tilde{\rho}'}{\partial z}\right) + \mathrm{RMS}\left(b'\right)}$$

Introducing the data assimilation capability

Design of the DA

- Type of DA: variational (currently 3DVar and 3DFGAT implemented)

$$J(\delta\mathbf{x}) = \frac{1}{2}\delta\mathbf{x}^{\mathrm{T}}\mathbf{B}^{-1}\delta\mathbf{x} + \frac{1}{2}\left(\mathbf{y}_t - \boldsymbol{\mathcal{H}}_t[\boldsymbol{\mathcal{M}}_t(\mathbf{x}^{\mathrm{b}})] - \mathbf{H}_t\mathbf{M}_t\delta\mathbf{x}\right)^{\mathrm{T}}\mathbf{R}^{-1}(\bullet)$$
$$\delta\mathbf{x} = \mathbf{x} - \mathbf{x}^{\mathrm{b}}.$$

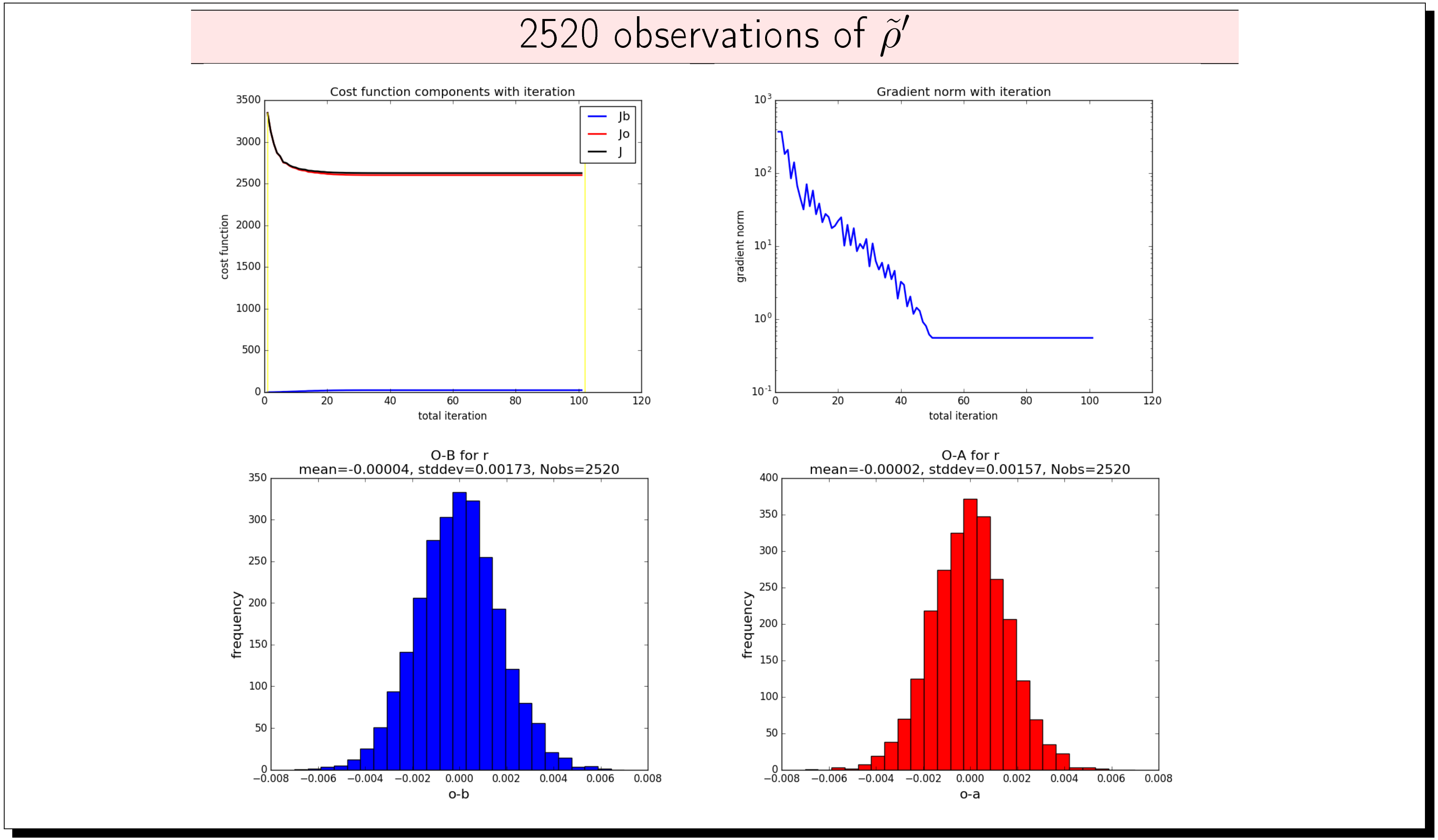
- Background error covariances ($\mathbf{B}$):
  - Need to be modelled efficiently (control variable transform, $\delta\mathbf{x} = \mathbf{U}\delta\boldsymbol{\chi}$, $\left\langle\delta\mathbf{x}\delta\mathbf{x}^{\mathrm{T}}\right\rangle = \mathbf{B}$, $\left\langle\delta\boldsymbol{\chi}\delta\boldsymbol{\chi}^{\mathrm{T}}\right\rangle = \mathbf{I}$),
  - Large range of options (e.g. exploiting geophysical balances, options for horizontal and vertical transforms).
- Observations:
  - Flexible observation network (e.g. any quantity, any time/position, allow outlier observations).

Introducing the data assimilation capability (cont.)

Design of the DA (cont.)

- Software needs to:
  - Calibrate $\mathbf{B}$-matrix from training data.
  - Test system components (e.g. adjoint tests, inverse tests).
  - Compute implied covariances: $\mathbf{B} = \mathbf{U}\mathbf{U}^{\mathrm{T}}$.
  - Compute raw covariances from training data $\left\langle \delta\mathbf{x}\delta\mathbf{x}^{\mathrm{T}} \right\rangle$.
  - Perform 'truth' run, $\mathbf{x}^{\mathrm{t}}$.
  - Generate $\mathbf{x}^{\mathrm{b}}$ by perturbing $\mathbf{x}^{\mathrm{t}}$.
  - Generate $\mathbf{y}_t$ by perturbing $\boldsymbol{\mathcal{H}}_t[\boldsymbol{\mathcal{M}}_t(\mathbf{x}^{\mathrm{t}})]$.
  - Assimilate!
  - Manage cycling.
  - Easy visualisation of assimilation diagnostics.
  - Compare assimilation experiments with different specifications.

Example diagnostics



Example investigation: geo. bal. in B-matrix

$$\delta \mathbf{x} = \mathbf{U}\delta\boldsymbol{\chi}$$

$$\begin{pmatrix} \delta\mathbf{u} \\ \delta\mathbf{v} \\ \delta\mathbf{w} \\ \delta\tilde{\boldsymbol{\rho}}' \\ \delta\mathbf{b}' \end{pmatrix} = \begin{pmatrix} \mathbf{0} & \partial_x & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \partial_y & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \alpha\mathbf{L}^{\mathrm{gb}} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \alpha\mathbf{L}^{\mathrm{hb}}\mathbf{L}^{\mathrm{gb}} & \mathbf{0} & \mathbf{0} & \mathbf{L}^{\mathrm{hb}} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{U}_{\mathrm{s}}^{\psi} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{U}_{\mathrm{s}}^{\chi} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{U}_{\mathrm{s}}^{w} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{U}_{\mathrm{s}}^{\mathrm{u}\tilde{\rho}'} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{U}_{\mathrm{s}}^{\mathrm{u}b'} \end{pmatrix} \begin{pmatrix} \delta\boldsymbol{\chi}^{\psi} \\ \delta\boldsymbol{\chi}^{\chi} \\ \delta\boldsymbol{\chi}^{w} \\ \delta\boldsymbol{\chi}^{\mathrm{u}\tilde{\rho}'} \\ \delta\boldsymbol{\chi}^{\mathrm{u}b'} \end{pmatrix}$$

E.g. $\delta\boldsymbol{\psi} = \mathbf{U}_{\mathrm{s}}^{\psi}\delta\boldsymbol{\chi}^{\psi}$, $\quad \delta\mathbf{u} = \partial_x\delta\boldsymbol{\psi}$

geo. bal. $\delta^{\mathrm{b}}\tilde{\boldsymbol{\rho}}' = \mathbf{L}^{\mathrm{gb}}\delta\boldsymbol{\psi}$, $\quad$ hydro. bal. $\delta\mathbf{b}' = \mathbf{L}^{\mathrm{hb}}\delta\tilde{\boldsymbol{\rho}}'$, $\quad$ geo. bal. switch $\alpha = 0, 1$

Would like to compute the implied covariances $\mathbf{B} = \mathbf{U}\mathbf{U}^{\mathrm{T}}$.

Example investigation: geo. bal. in B-matrix (cont.)

$$\mathbf{B} = \mathbf{U}\mathbf{U}^{\mathrm{T}} = \begin{pmatrix} \partial_x \mathbf{B}^{\chi} \partial_x^{\mathrm{T}} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \partial_y \mathbf{B}^{\psi} \partial_y^{\mathrm{T}} & \mathbf{0} & \alpha \partial_y \mathbf{B}^{\psi} {\mathbf{L}^{\mathrm{gb}}}^{\mathrm{T}} & \alpha \partial_y \mathbf{B}^{\psi} {\mathbf{L}^{\mathrm{gb}}}^{\mathrm{T}} {\mathbf{L}^{\mathrm{hb}}}^{\mathrm{T}} \\ \mathbf{0} & \mathbf{0} & \mathbf{B}^{w} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \alpha \mathbf{L}^{\mathrm{gb}} \mathbf{B}^{\psi} \partial_y^{\mathrm{T}} & \mathbf{0} & \mathbf{B}^{\tilde{\rho}'} & \mathbf{B}^{\tilde{\rho}'} {\mathbf{L}^{\mathrm{hb}}}^{\mathrm{T}} \\ \mathbf{0} & \alpha \mathbf{L}^{\mathrm{hb}} \mathbf{L}^{\mathrm{gb}} \mathbf{B}^{\psi} \partial_y^{\mathrm{T}} & \mathbf{0} & \mathbf{L}^{\mathrm{hb}} \mathbf{B}^{\tilde{\rho}'} & \mathbf{L}^{\mathrm{hb}} \mathbf{B}^{\tilde{\rho}'} {\mathbf{L}^{\mathrm{hb}}}^{\mathrm{T}} + \mathbf{B}^{\mathrm{u}b'} \end{pmatrix} \begin{matrix} \leftarrow u \\ \leftarrow v \\ \leftarrow w \\ \leftarrow \tilde{\rho}' \\ \leftarrow b' \end{matrix}$$

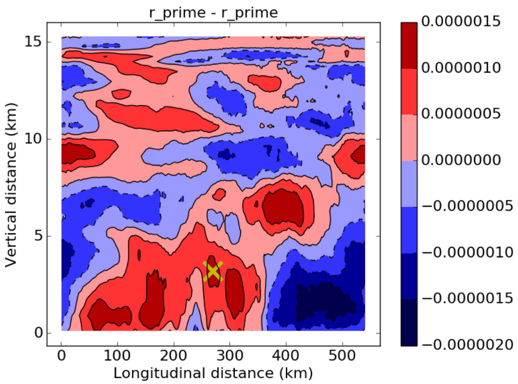
$$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ u & v & w & \tilde{\rho}' & b' \end{matrix}$$

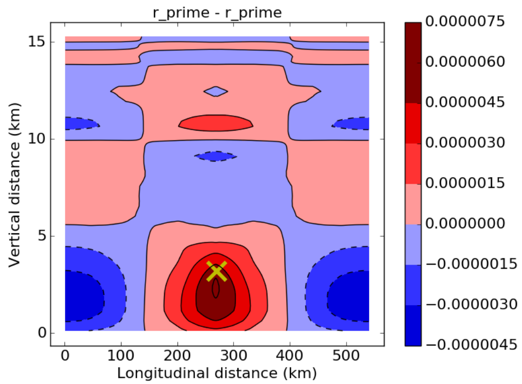
where $\mathbf{B}^{\tilde{\rho}'} = \alpha^2 \mathbf{L}^{\mathrm{gb}} \mathbf{B}^{\psi} {\mathbf{L}^{\mathrm{gb}}}^{\mathrm{T}} + \mathbf{B}^{\mathrm{u}\tilde{\rho}'}$, and $\mathbf{B}^{\psi} = \mathbf{U}_{\mathrm{s}}^{\psi} {\mathbf{U}_{\mathrm{s}}^{\psi}}^{\mathrm{T}}$, etc.

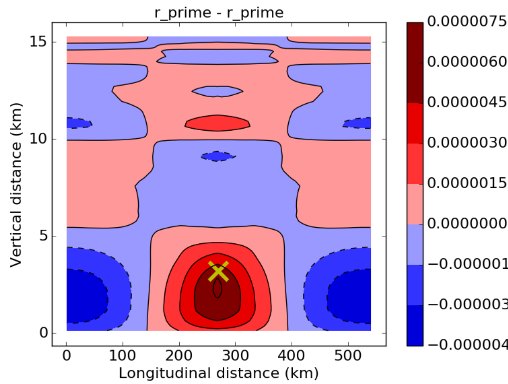
Example investigation: geo. bal. in B-matrix (cont.)

	Cov. stats $\langle \delta\mathbf{x}\delta\mathbf{x}^{\mathrm{T}} \rangle$	Implied Cov., $\mathbf{U}\mathbf{U}^{\mathrm{T}}$ (GB+)	Implied Cov., $\mathbf{U}\mathbf{U}^{\mathrm{T}}$ (GB-)

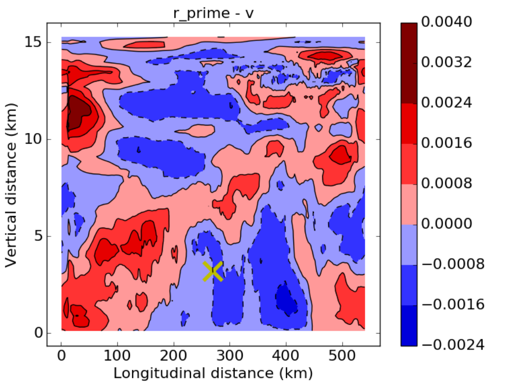
$\tilde{\rho}' - \tilde{\rho}'$

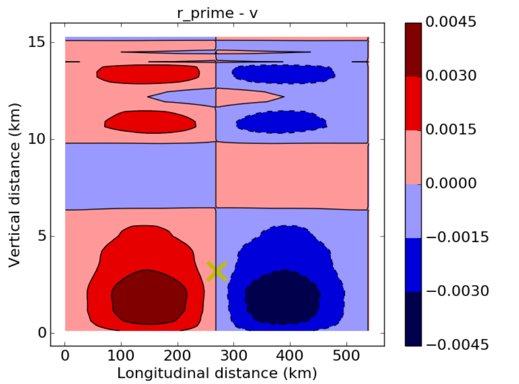


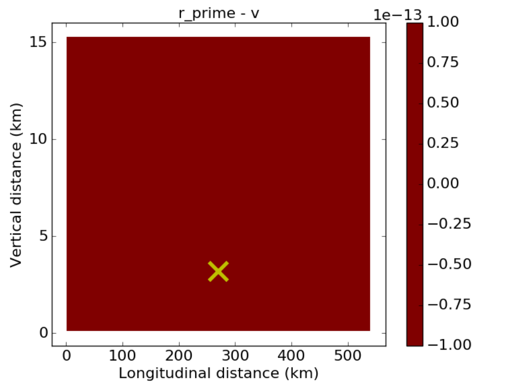




$v' - \tilde{\rho}'$

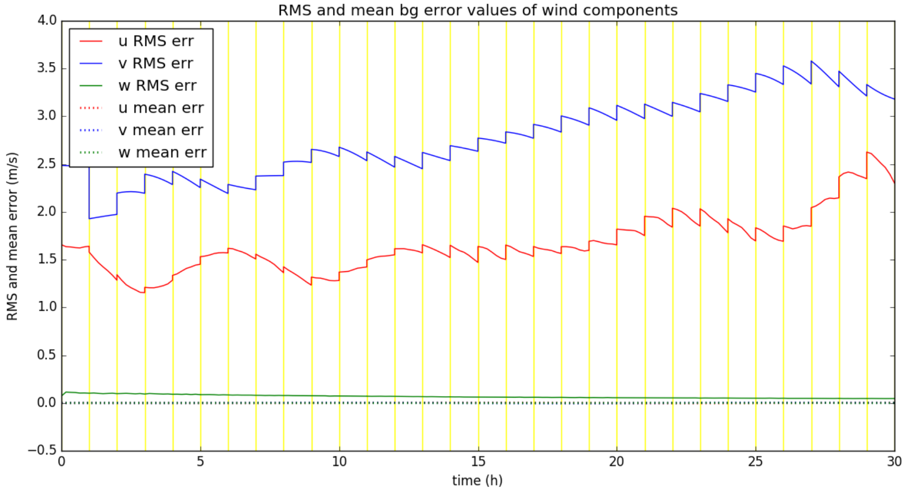




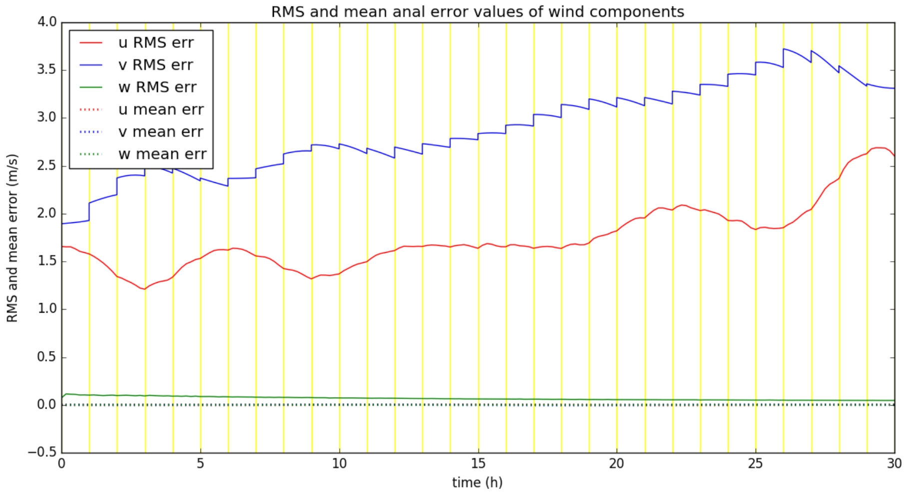


Multi-cycle twin exps (2520 obs of $\tilde{\rho}'$): with geo. bal.

$\alpha = 1$ (GB+), background wind errors

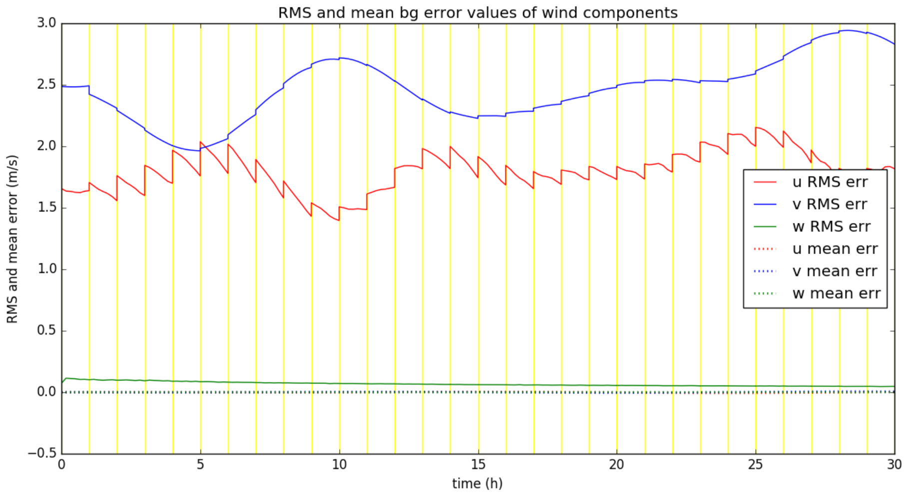


$\alpha = 1$ (GB+), analysis wind errors

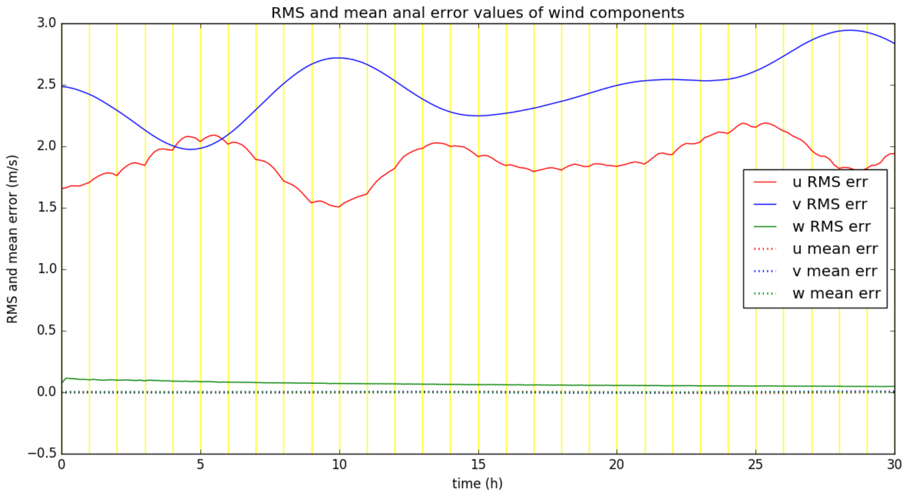


Multi-cycle twin exps (2520 obs of $\tilde{\rho}'$): without geo. bal.

$\alpha = 0$ (GB-), background wind errors

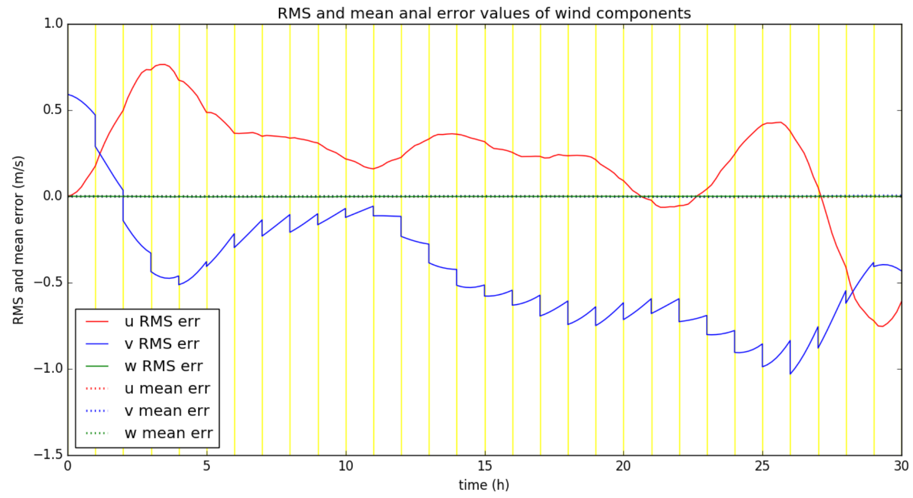


$\alpha = 0$ (GB-), analysis wind errors



Multi-cycle twin exps (2520 obs of $\tilde{\rho}'$): which is better?

background wind RMSE differences



$$\text{diff} = \text{RMSE(GB-)} - \text{RMSE(GB+)}$$

background scaled density RMSE differences

Negative values indicate that removing geo. bal. operators from $\mathbf{B}$ (i.e. $\alpha = 0$) improves the assimilation.

Summary

- Have built a functioning, flexible, and fast variational DA system around a simplified 2D convective-scale model.
- Taster:
  - Observations of $\tilde{\rho}'$.
  - Switch ON/OFF geostrophic constraint in $\mathbf{B}$-matrix.
  - Same initial background state.
  - Cycle length 1 hour, 30 cycles.
  - Twin experiments: know truth.
  - Mixed impact of switching off geostrophic balance – often reduces RMS errors.
- Future:
  - Publish system and tests.
  - Investigate other balances.
  - Add: 4D-Var, moisture.
  - Normal mode-based $\mathbf{B}$-matrix.
  - Reduced-rank Kalman Filter . . .