A revised version of the ‘modRSW’ model to facilitate satellite DA research

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Researching satellite data assimilation

- Satellite observations are an essential ingredient in current data assimilation systems.
- They have greatly contributed to the improvement of weather forecasts over time.
- New and more precise instruments boarded on satellites are added every year to the observing system.

Aim: to utilise an idealised model - i.e. ‘modRSW’ - to help investigate the impact of satellite observations in a DA system: what is the impact of large-scale or small-scale?

Modelling satellite observations

Idealised satellite DA will require the generation of (synthetic) satellite observations.
Our focus is on **sounding observations**.

Satellite observations	Current modRSW setup
Radiance (via Brightness Temperature)	✔(*)
Vertical structure	✗ single-layer
Spatially varying	✗ fixed in space
Non-linear observation operator	✗ linear

(*) scaling issue, see next slide

Scaling for ‘modRSW’ in presence of temperature

- We tried to define a diagnostic equation for temperature based on hydrostatic equilibrium and the ideal gas law:

$$T = \frac{gh}{R}$$

- The scaling for gH used in [1] ($gH=330m^2s^{-2}$) leads to values of temperature of order O(1) K. But that was chosen to maintain the Froude number above 1 (with U=20m/s):

$$\mathrm{Fr} = \frac{U}{\sqrt{gH}}$$

A reminder: Fr>1 implies supercritical regime which implies traveling gravity waves (i.e. convection moving across the domain)

[1] Kent, T. et al (2017): *Dynamics of an idealized fluid model for investigating convective-scale data assimilation*. Tellus A: Dynamic Meteorology and Oceanography, 69(1), 1369332.

Modelling satellite observations

Idealised satellite DA will require the generation of (synthetic) satellite observations. Our focus is on **sounding observations**.

Satellite observations	Current modRSW setup	Revised modRSW setup
Radiance (via Brightness Temperature)	✔(*)	✔
Vertical structure	✗ single-layer	✔
Spatially varying	✗ fixed in space	✔
Non-linear observation operator	✗ linear	✔

(*) scaling issue, see next slide

Modifications

The modifications to the current modRSW setup will concern three aspects:

- The mathematical formulation;
- The way the synthetic observations are generated from the truth, separating satellite observations from ground observations;
- The observation operator $\mathscr{H}$ which maps the model state into the observational space.

The revised model

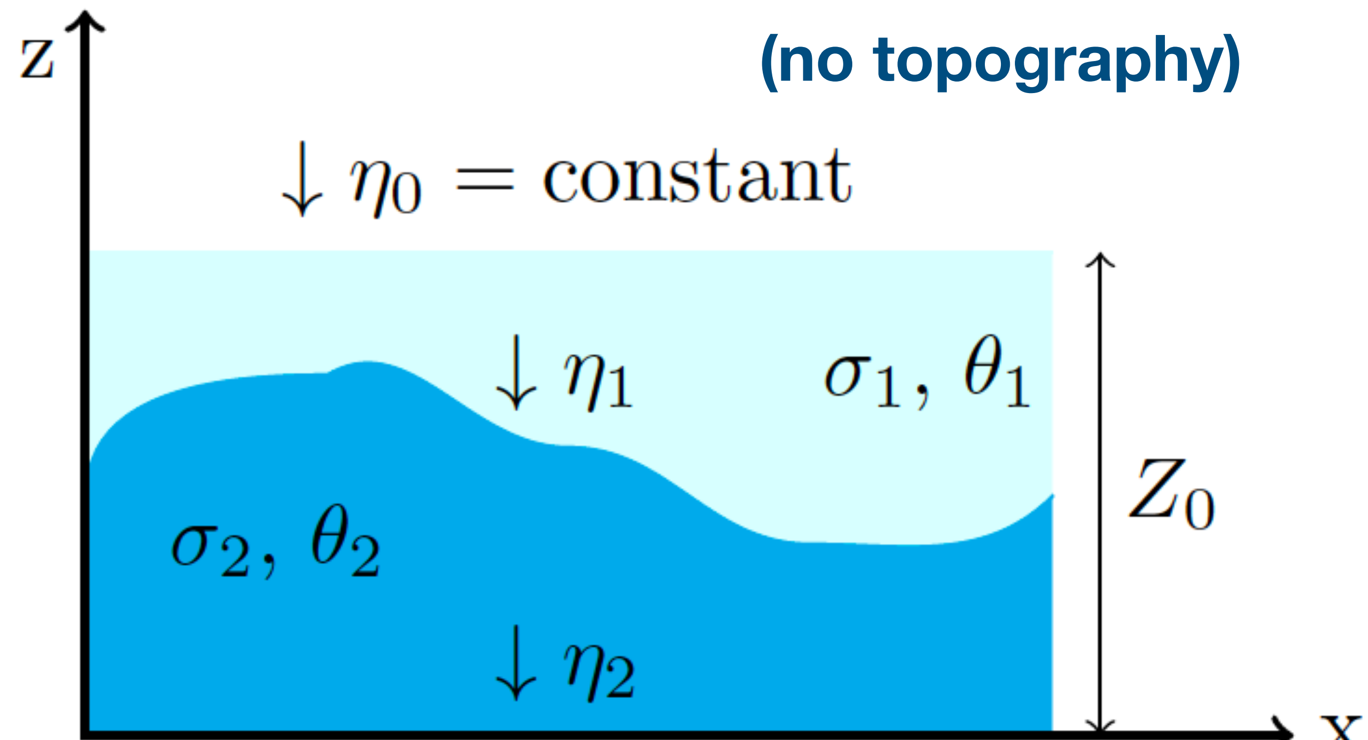
Two assumptions:

- **isentropic** fluid (robust definition of temperature):

$$T_i = \theta_i \eta_i^{\kappa} \qquad \boxed{\eta_i = \frac{p_i}{p_r}}$$

Pressure (non-dim)

- two layers of fluid in which **the one on the top is inactive** - $u_1=0$ - and capped by a **rigid lid** (i.e. **1.5 layer**).



$$\boxed{\sigma_i = \frac{p_r}{g}\left(\eta_{i+1} - \eta_i\right)}$$

Pseudo-density (replaces h)

N.B. We can still solve just one set of equations (for the bottom layer).

$$\left(h, hu, hv, hr\right) \rightarrow \left(\sigma_2, \sigma_2 u, \sigma_2 v, \sigma_2 r\right)$$

The revised model

The new full set of equations reads as:

$$
\begin{aligned}
(\sigma_2)_t + \left(\sigma_2 u_2\right)_x &= 0, \\
\left(\sigma_2 u_2\right)_t + \left(\sigma_2 u_2^2 + \mathscr{M}(\eta_2)\right)_x + c_0^2 \sigma_2 r_x - f\sigma_2 v_2 &= 0, \\
\left(\sigma_2 v_2\right)_t + (\sigma u v)_x + f\sigma u &= 0, \\
\left(\sigma_2 r\right)_t + \left(\sigma_2 u_2 r\right)_x + \beta\sigma_2 (u_2)_x + \alpha\,\sigma_2 r &= 0\,.
\end{aligned}
$$

$$
\mathscr{M}(\eta_2) = \begin{cases} \mathscr{M}(\eta_2(\sigma_2)) & \text{if } \sigma < \sigma_c, \\ \mathscr{M}_c(\eta_c(\sigma_c)) & \text{if } \sigma \geq \sigma_c, \end{cases}
$$

$$
\beta = \begin{cases} \tilde{\beta} & \text{if } \sigma \geq \sigma_r, \\ 0 & \text{otherwise}\,. \end{cases}
$$

Black = classic shallow water w/rotation
Red = shallow water with convection/rain

- The pseudo density is a non-linear function of the non-dim pressure η_2:

$$
\sigma_2 = \eta_2 - \left(\frac{\theta_2}{\Delta\theta}\right)^{\frac{1}{\kappa}} \left(-\eta_2^\kappa + \frac{\theta_1}{\theta_2}\eta_0^\kappa + \frac{gZ_0}{c_p\theta_2}\right)^{\frac{1}{\kappa}}
$$

This function is inverted online to obtain η

Checks against an analytical solution

- We derived an ODE for v from the shallow water system (**without** convection and precipitation) for stationary waves (after having defined ξ=x-ct, see Shrira papers [2],[3]):

$$v'' = \frac{1}{\mathrm{Ro}^2} \frac{v}{\tilde{c}_p \theta_2 \kappa \eta^{\kappa-1} \left(\frac{1}{\frac{\partial \sigma}{\partial \eta}} \right) \sigma_0 - \frac{1}{\mathrm{Ro}^3} \frac{1}{\left(\frac{1}{\mathrm{Ro}} + v' \right)}}$$

$$\sigma = \sigma_0 (\mathrm{Ro} + v')$$

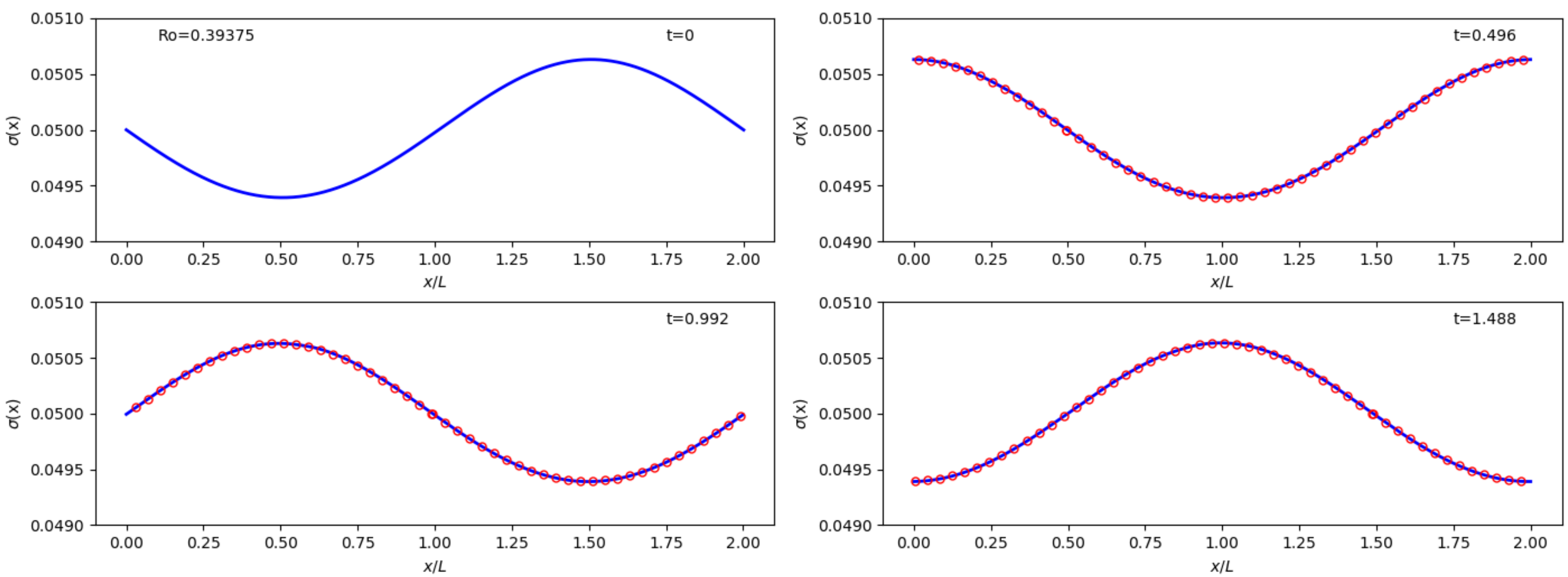
$$u = \frac{v'}{\frac{1}{\mathrm{Ro}} + v'}$$

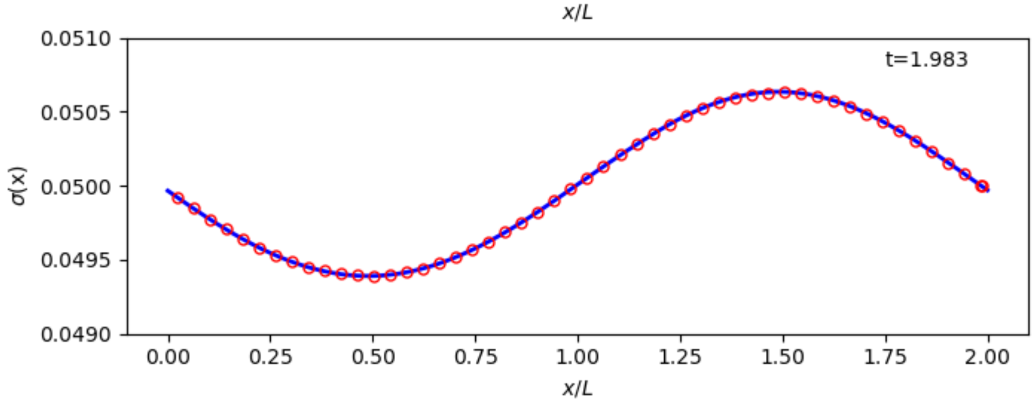
- We compared the solution of this equation (a stationary wave) translated in time against its evolution predicted by the numerical model (in a periodic domain).

[2] Shrira, V. (1981), *Propagation of long nonlinear waves in a layer of rotating fluid*, Sov. Phys. - Izvestija, vol. 17, n. 1, pp 55-59.
[3] Shrira, V. (1986), *On the long strongly nonlinear waves in rotating ocean*, Sov. Phys. - Izvestija, vol. 22, n. 4, pp. 285-305.

Checks against an analytical solution

$$v'' = \frac{1}{\mathrm{Ro}^2}\frac{v}{\tilde{c}_p \theta_2 \kappa \eta^{\kappa-1}\left(\frac{1}{\frac{\partial \sigma}{\partial \eta}}\right)\sigma_0 - \frac{1}{\mathrm{Ro}^3}\frac{1}{\left(\frac{1}{\mathrm{Ro}} + v'\right)}} \qquad \sigma = \sigma_0\left(\mathrm{Ro} + v'\right) \qquad u = \frac{v'}{\frac{1}{\mathrm{Ro}} + v'}$$





Blue line: numerical solution

Red circles: stationary wave translated in time

Idealised satellite observations

The radiative scheme

- Synthetic observations of radiance B are generated using the Rayleigh-Jeans law (valid for λ>50μm at T=300K):

$$B = 2\frac{k_B c}{\lambda^4}T = 2\frac{k_B c}{\lambda^4}\theta\eta^\kappa \rightarrow B' = \frac{B}{B_0} = \eta^\kappa$$

Spatially varying observations

- Let's consider polar-orbit satellites: they move and observe different portions of the Earth at different times.

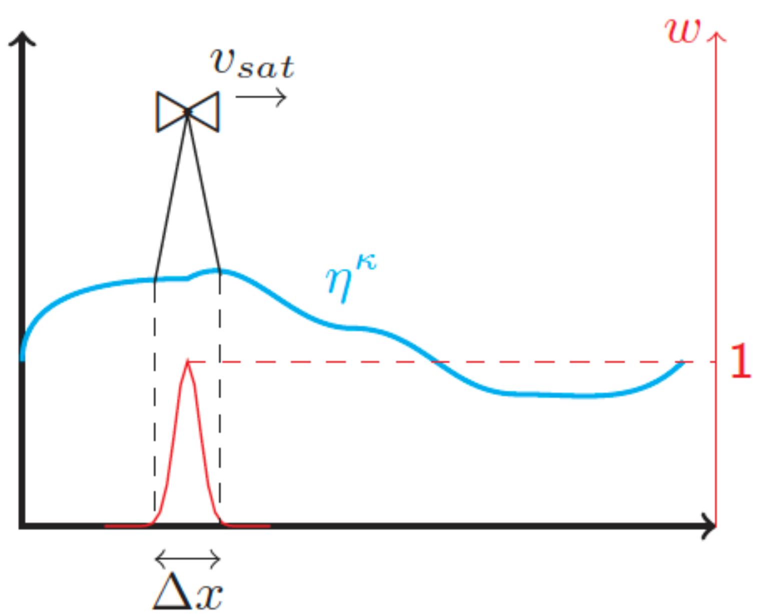
- 1 D approximation: our satellite observations move with velocity v_{sat} along a periodic domain of length L:

$$x_{sat} = v_{sat} \cdot t \mod L$$

Idealised satellite observations

Horizontally-averaged observations

- To mimic the satellite's Field Of View (FOV), a weighted mean is applied to a Δx window:



$$y^o(x_{sat}) = \int_{x_{sat}-\frac{\Delta x}{2}}^{x_{sat}+\frac{\Delta x}{2}} w(x)\eta^{\kappa}\mathrm{dx}$$

- w(x) is a Gaussian function centred on x_{sat} which is as wide as Δx.

N.B. All this is done only for σ. The other variables (u,v,r) are observed as before.

A new observation operator

- The new observation vector is split into satellite and ground observations:

$$\mathrm{y}^o = \begin{pmatrix} y^o(x_{sat}) \\ \mathrm{y}^o(\mathrm{x}_{grn}) \end{pmatrix} = \begin{pmatrix} (\eta_2^t)^\kappa(x_{sat}) \\ \mathrm{y}^o(\mathrm{x}_{grn}) \end{pmatrix},$$

in which the ground observations y^{o}_{grn} are direct observations of u,v,r at fixed x_{grn} positions along the domain.

- The new observation operator $\mathscr{H}$ reads as:

$$\mathscr{H}(\mathbf{x}^f) = \begin{pmatrix} y^f(x_{sat}) \\ \mathrm{y}^f(\mathrm{x}_{grn}) \end{pmatrix} = \begin{pmatrix} (\eta_2^f)^\kappa(x_{sat}) \\ \mathrm{y}^f(\mathrm{x}_{grn}) \end{pmatrix}.$$

What happens to the EnKF?

- We made no changes to the DA scheme, still an EnKF:

$$x^a = x^f + K\left(y^o - \mathscr{H}(x^f)\right) \qquad K = P^f H^T \left(HP^f H^T + R\right)^{-1}$$

- A common way of using an EnKF in the presence of a non-linear observation operator is given by Houtekamer & Mitchell (see [5]):

$$P^f H^T = \frac{1}{N-1}\sum_{i=1}^{N}\left(x^f - \overline{x^f}\right)\left(\mathscr{H}x^f - \overline{\mathscr{H}x^f}\right)^T, \qquad \overline{x^f} = \frac{1}{N}\sum_{i=1}^{N} x_i^f$$

$$HP^f H^T = \frac{1}{N-1}\sum_{i=1}^{N}\left(\mathscr{H}x^f - \mathscr{H}\overline{x^f}\right)\left(\mathscr{H}x^f - \overline{\mathscr{H}x^f}\right)^T \qquad \overline{\mathscr{H}x^f} = \frac{1}{N}\sum_{i=1}^{N} \mathscr{H}x_i^f$$

This, though, is hard to combine with the model-space localisation used in current modRSW setup.

[5] Houtekamer, P. L., & Mitchell, H. L. (2001). A sequential ensemble Kalman filter for atmospheric data assimilation. *Monthly Weather Review*, *129*(1), 123-137.

What happens to the EnKF?

- Instead, we decided to linearise the observation operator $\mathscr{H}$ only for the purpose of computing the Kalman Gain:

$$H \simeq \partial_{x^f}\mathscr{H} = \left(\partial_{\sigma_2^f}\mathscr{H}, \partial_{u^f}\mathscr{H}, \partial_{v^f}\mathscr{H}, \partial_{r^f}\mathscr{H}, \right)$$

- This assumption of course is not optimal (even if we don't know how deleterious it is), but at this stage it's less time-consuming than moving from model-space localisation to observation-space localisation.

[5] Houtekamer, P. L., & Mitchell, H. L. (2001). A sequential ensemble Kalman filter for atmospheric data assimilation. *Monthly Weather Review*, *129*(1), 123-137.

Conclusions

- We have modified the single-layer isopycnal ‘modRSW’ into an isentropic 1.5-layer model. We checked the new model (without convection and precipitation) against an analytical solution.

- The observations are now split into satellite and ground ones. Satellite observations are modelled as radiance measurements which take into account both the spatially varying character of polar-orbit satellite and are averaged horizontally to mimic the FOV.

- We modified the observation operator accordingly, into a new, non-linear one.

Future work

- Modify the model in order to include topography.
- Explore the possibility of defining weighting functions and using a multi-channel approach in assimilating radiance.
- Find the best strategy to define clouds.
- Explore alternative radiation schemes.
- Ultimately, use the new setup to investigate the relative impact of observing large-scale and small-scale features.