

(MAD)*DAM
Moist Atmosphere Dynamics
Data Assimilation Model:
Application to 4D-Var Wind Tracing

University of Ljubljana
Faculty of *Mathematics and Physics*



Žiga Zaplotnik¹, Nedjeljka Žagar²

¹University of Ljubljana, Faculty of Mathematics and Physics

²University of Hamburg, Faculty of Mathematics, Informatics and Natural Sciences

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Introduction

- Simplified models **allow to**:
 - Develop and implement new algorithms faster than in the NWP case
 - Perform numerical experiments in a controlled environment in which various issues, difficult to grasp in a real NWP, can more easily be understood
- Simplified models **should be**:
 - complex enough to capture main physical aspects of the phenomena of interest in order to explain the observed features of atmospheric circulation and thus to be of any value for NWP

Introduction

- The new intermediate-complexity system (MADDAM) was developed to study:
 - Interaction of gravity waves and moisture in 4D-Var
 - Coupling of the aerosols-moisture-winds in 4D-Var (wind tracing)
- Two components of the system:
 - Moist Atmosphere Dynamics (MAD) forecast model
 - Moist Atmosphere Dynamics Data Assimilation Model (MADDAM)

MAD forecast model

- Single-vertical level spectral prognostic model for the Tropics (based on Gill (1980,1982), Davey and Gill (1987), built upon Žagar (2008,2012))
- Simulates **nonlinear** interactions between condensational heating and dynamics
- Prescribed baroclinic (vertical wavenumber $m=1$) vertical structure with sinusoidal profile of heating:

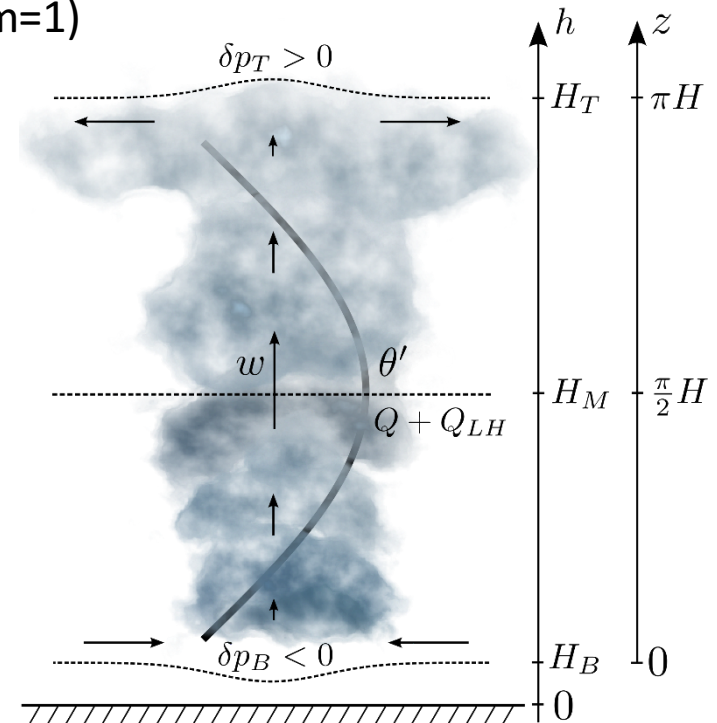
$$Q_{LH} = \tilde{Q}_{LH}(x, y, t) \sin\left(\frac{z}{H}\right) \frac{\theta_0(z)}{\theta_{00}}$$

$$\theta' = \tilde{\theta}'(x, y, t) \sin\left(\frac{z}{H}\right) \frac{\theta_0(z)}{\theta_{00}}$$

$$w = \tilde{w}(x, y, t) \sin\left(\frac{z}{H}\right)$$

$$\mathbf{v}_h = \tilde{\mathbf{v}}_h(x, y, t) \cos\left(\frac{z}{H}\right)$$

$$p' = \tilde{p}' \cos\left(\frac{z}{H}\right) \frac{\rho_0(z)}{\rho_{00}}$$



MAD forecast model

Prognostic equations

$$\frac{\partial \theta}{\partial t} + (\mathbf{v} \cdot \nabla) \theta - \frac{\theta_{00} N^2 H}{g} (\nabla \cdot \mathbf{v}) = \frac{L_c}{2 c_p} C + Q - \varepsilon_\theta \theta$$

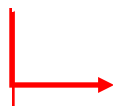
Potential temp. pert. at
midtropospheric level (~ 400 hPa)
Thermodynamic + continuity eq.

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + f \mathbf{k} \times \mathbf{v} = \frac{gH}{\theta_{00}} \nabla \theta - \varepsilon_v \mathbf{v}$$

Winds at lower level (~ 700 hPa)

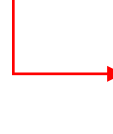
$$\frac{\partial q}{\partial t} + \nabla \cdot (\mathbf{v} q) = E - C$$

Specific humidity [g/kg]
at lower level



$$C = -\nabla \cdot (\mathbf{v} q) \text{ if } q \geq q_s(\theta)$$

Condensation due to large-scale convergence
Excess humidity is immediately released



$$q_s(\theta) = \frac{e_s(T(\theta)) R_d}{p R_v}$$

Temperature-dependent
saturation specific humidity

$$\frac{\partial c}{\partial t} + \nabla \cdot (\mathbf{v} c) = -K_w c P - K_d c + S^+ - S^-$$

Total aerosol mass mixing ratio
[$\mu\text{g}/\text{kg}$] at lower level

MAD forecast model

Model parameters

- Tropopause height: $\pi H = 15$ km
- Lapse rate: $d\theta_0/dz = 4$ K km⁻¹
- Mid-level mean potential temperature: $\theta_{00} = 333$ K
- Bouyancy frequency: $N = 0.011$ s⁻¹, horizontal phase speed: $c = NH = 50$ m/s
- Deformation radius: $R_e = \sqrt{c/(2\beta)} = 1000$ km, adjustment time scale: $T = R_e/c = 6$ h

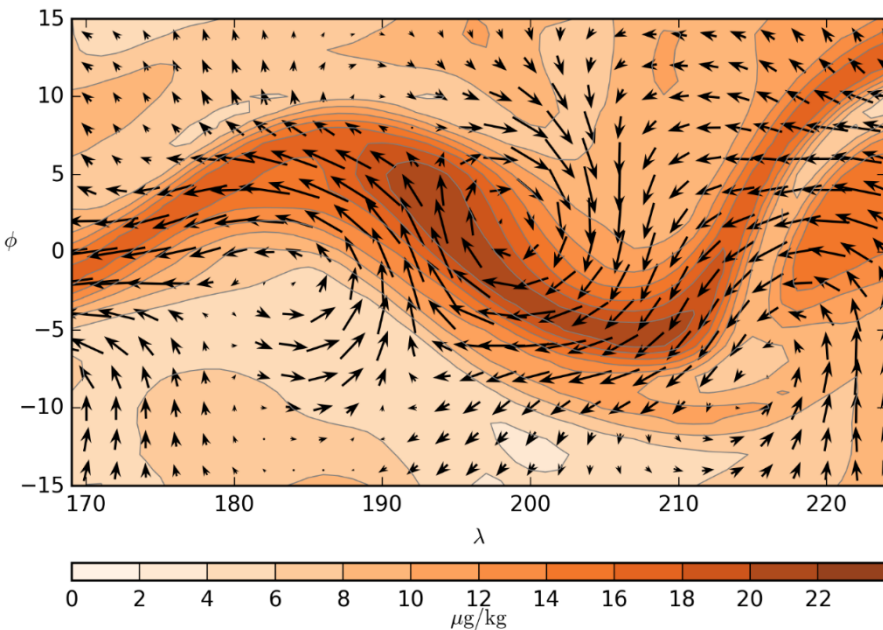
Numerical formulation

- Spectral model, $N_k=119$, $N_l=23$, a total of 2215 2D Fourier waves after elliptic truncation, quadratic grid, 4th order implicit numerical diffusion
- Leap-frog time-stepping, $\Delta t=180$ s, Asselin filtering
- Domain: tropical belt spanning from 32°S to 32°N, resolution 1°x1°
- Boundary relaxation zone 6° wide, extension zone 7° wide
- Mass fixers to correct negative values due to spectral truncation for positive definite fields, e.g. aerosol, humidity

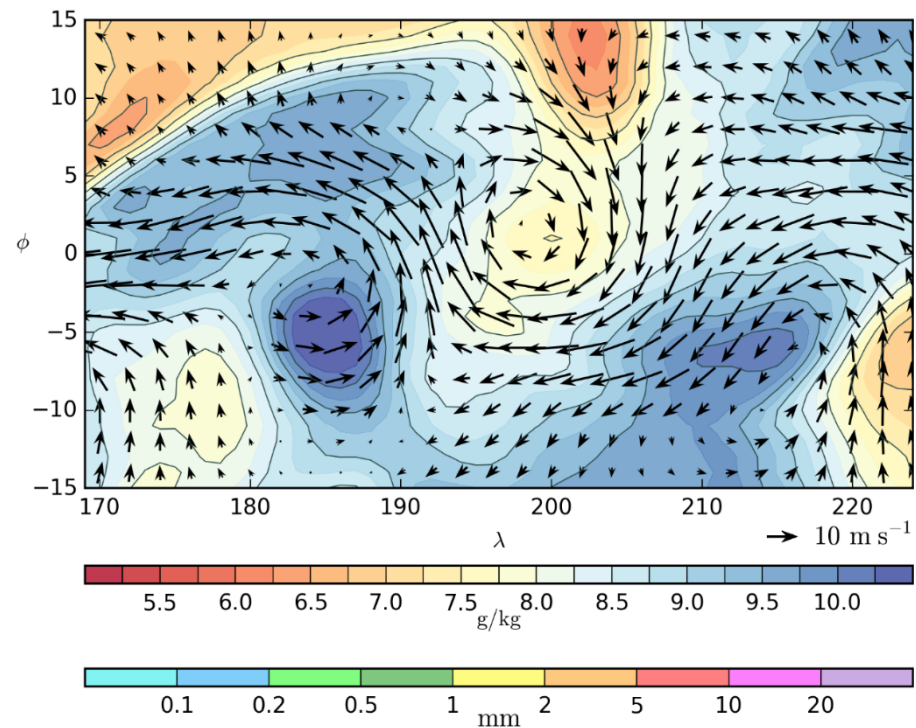
Aerosol-moisture-wind coupling in forecast model

00

Total aerosol mass mixing ratio



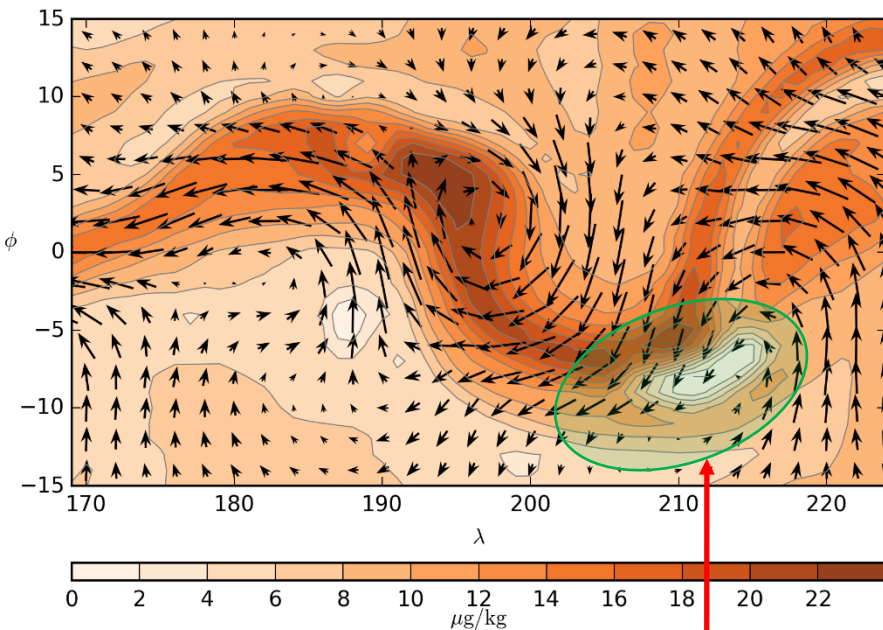
Specific humidity, 6-hour cum. precipitation



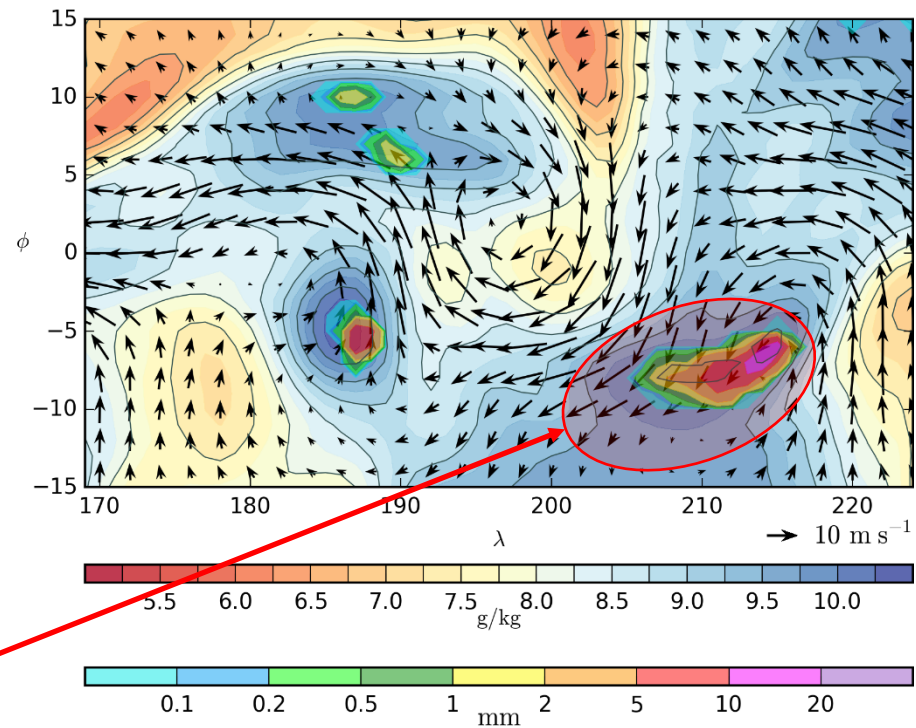
Aerosol-moisture-wind coupling in forecast model

+06 h

Total aerosol mass mixing ratio



Specific humidity, 6-hour cum. precipitation

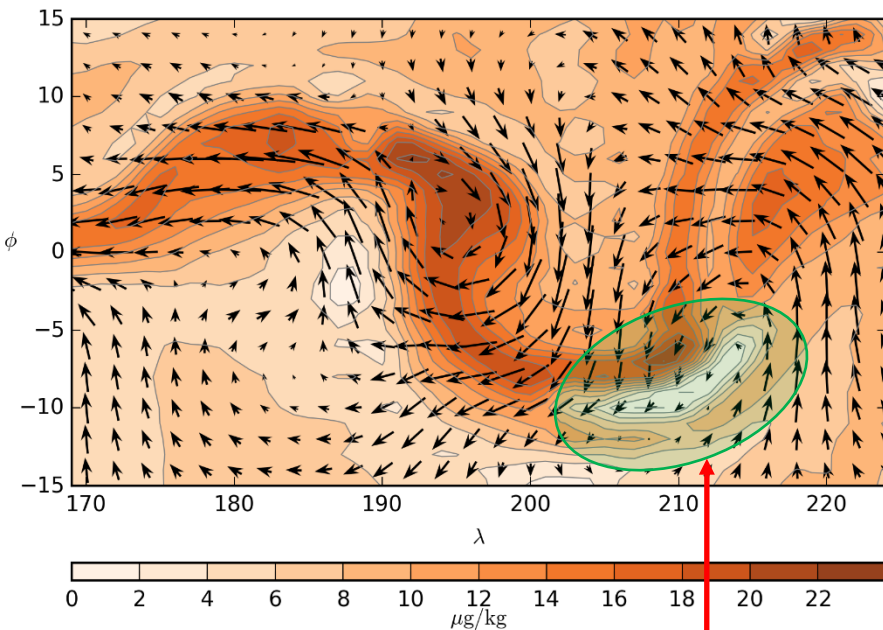


Wet deposition of aerosol

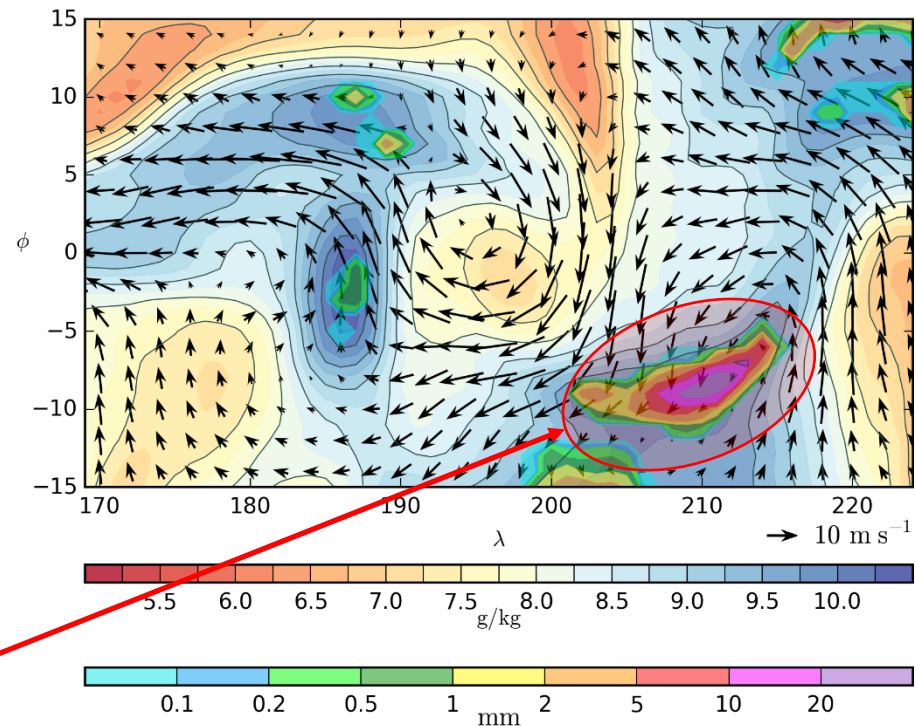
Aerosol-moisture-wind coupling in forecast model

+12 h

Total aerosol mass mixing ratio



Specific humidity, 6-hour cum. precipitation

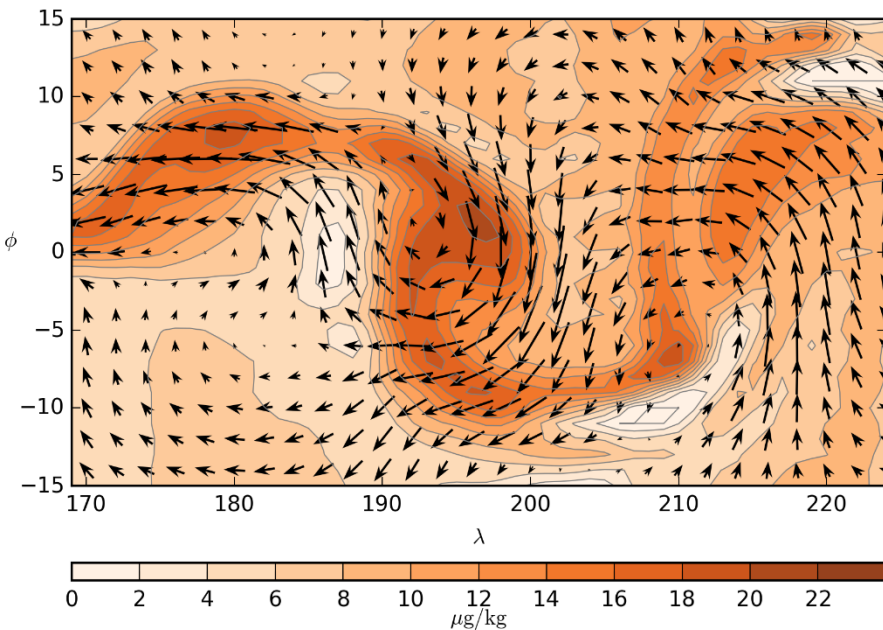


Wet deposition of aerosol

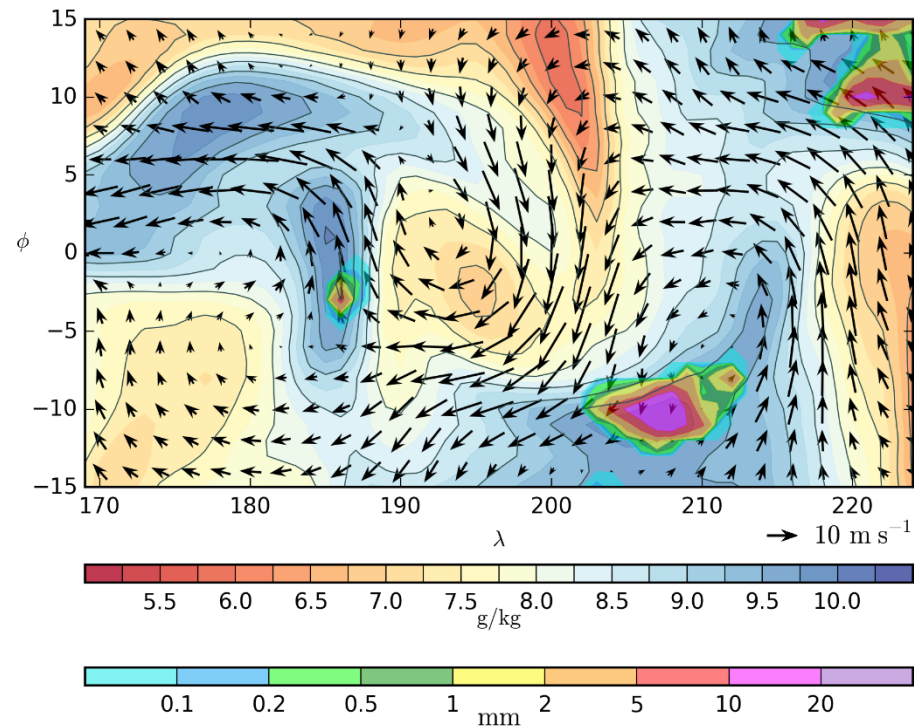
Aerosol-moisture-wind coupling in forecast model

+18 h

Total aerosol mass mixing ratio



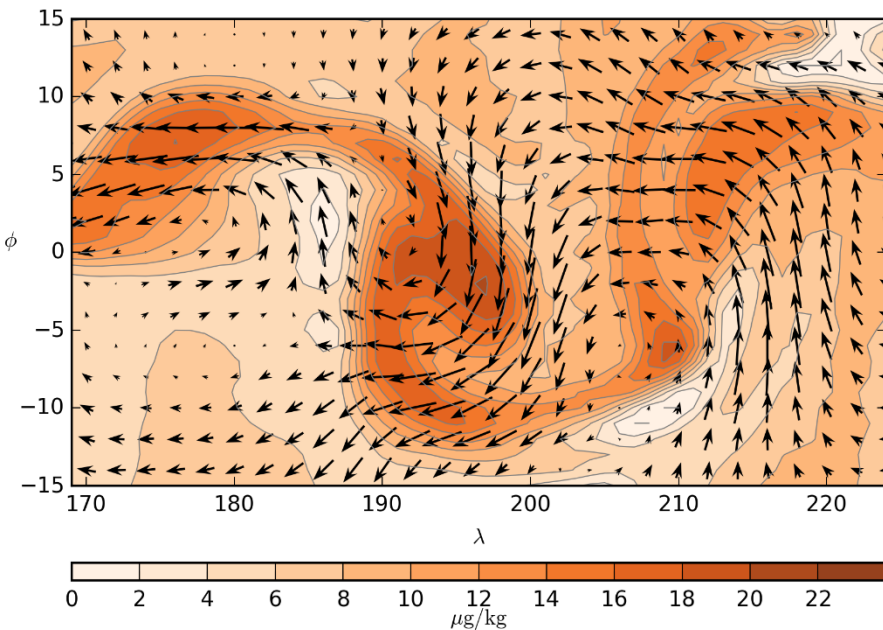
Specific humidity, 6-hour cum. precipitation



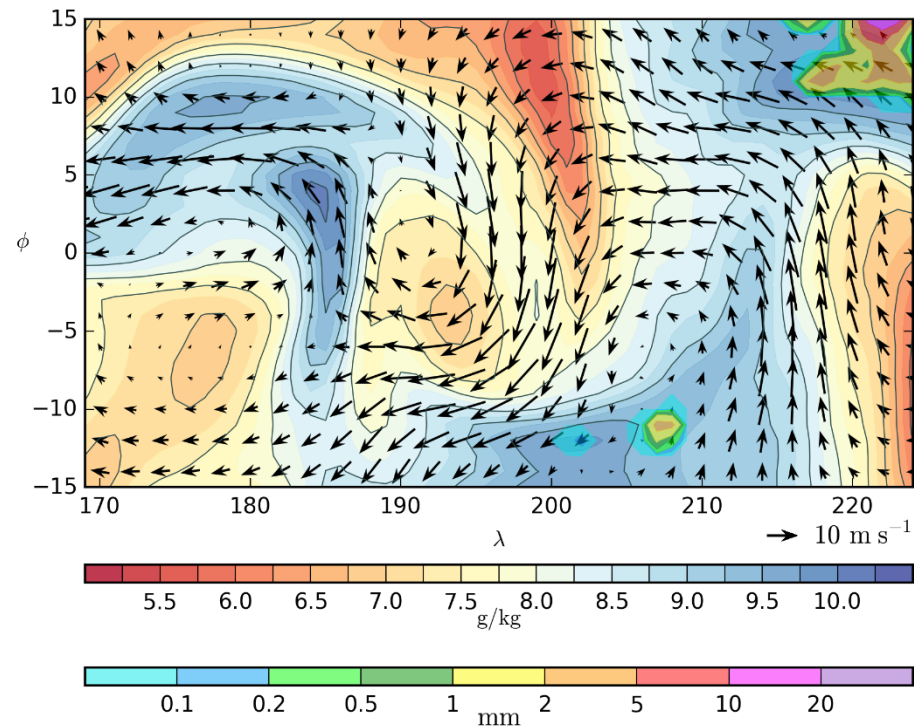
Aerosol-moisture-wind coupling in forecast model

+24 h

Total aerosol mass mixing ratio



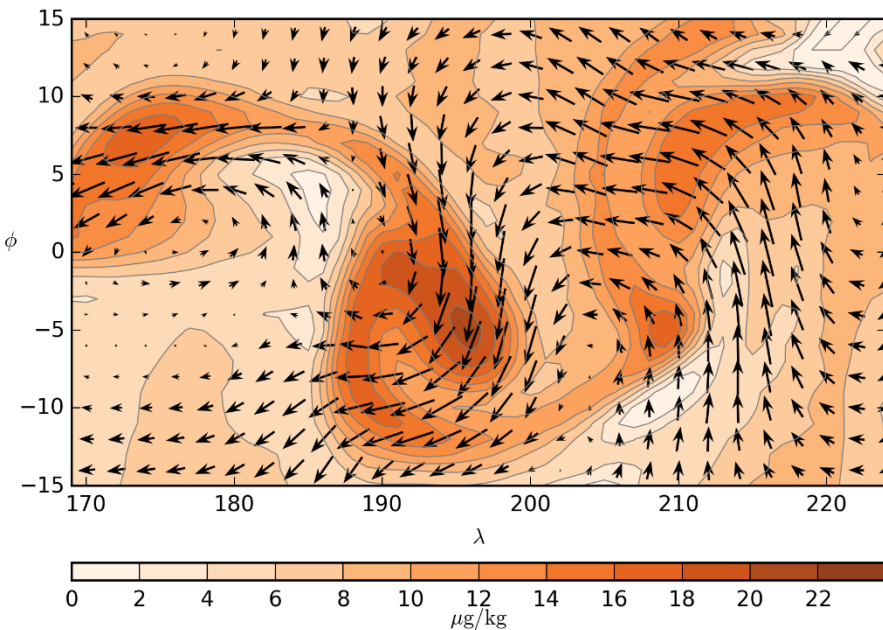
Specific humidity, 6-hour cum. precipitation



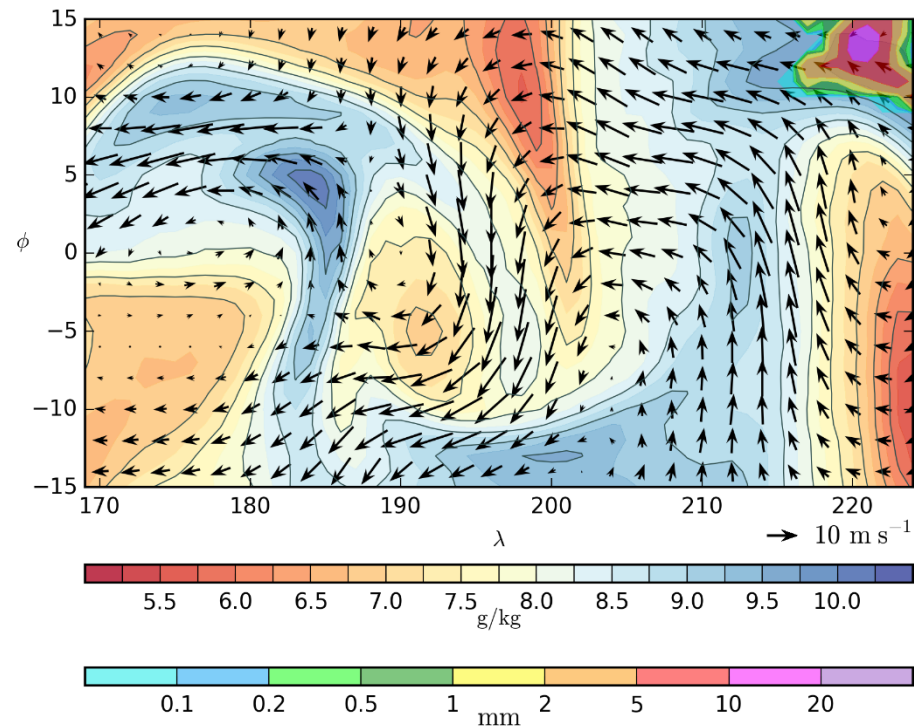
Aerosol-moisture-wind coupling in forecast model

+30 h

Total aerosol mass mixing ratio



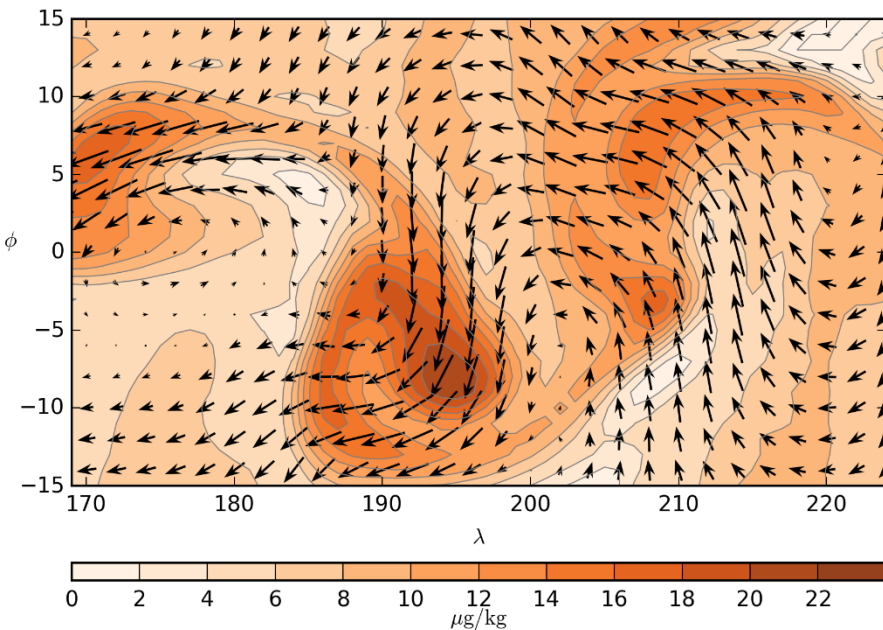
Specific humidity, 6-hour cum. precipitation



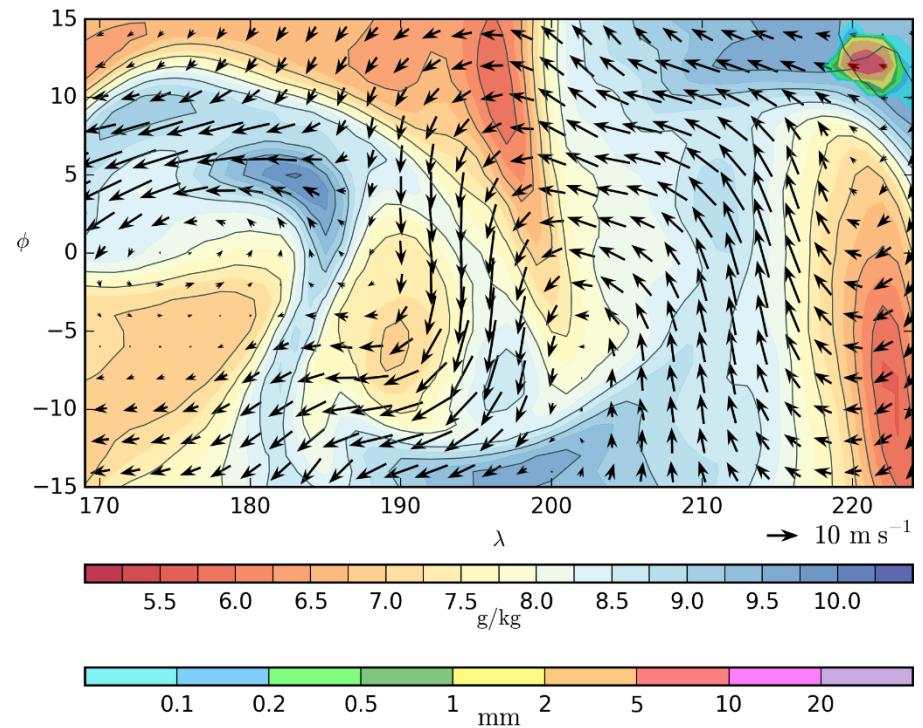
Aerosol-moisture-wind coupling in forecast model

+36 h

Total aerosol mass mixing ratio



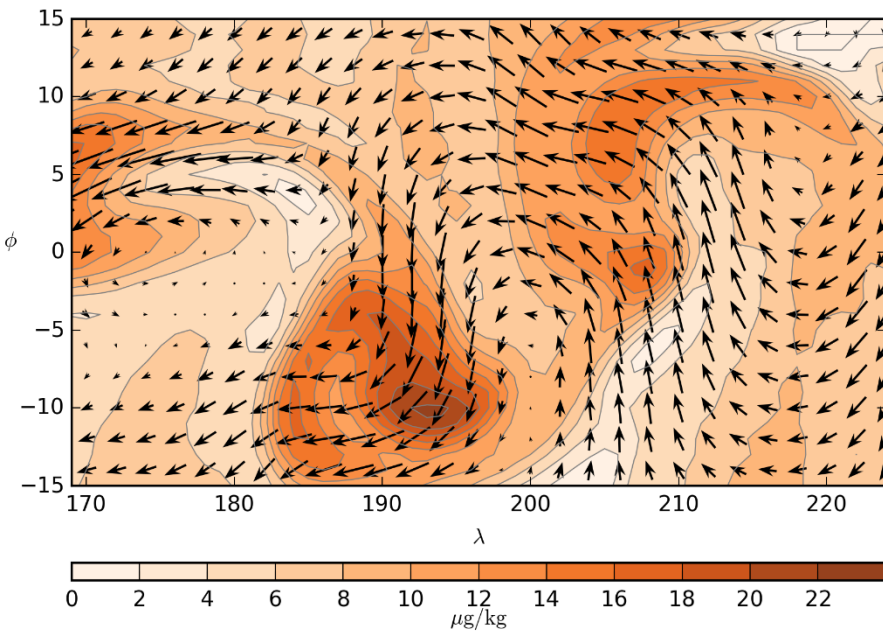
Specific humidity, 6-hour cum. precipitation



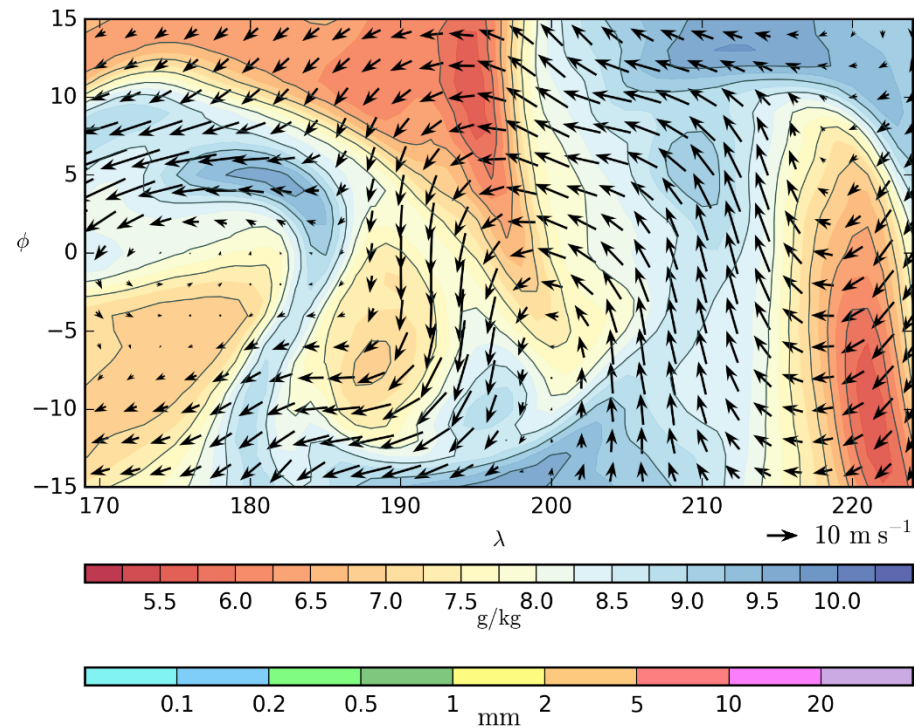
Aerosol-moisture-wind coupling in forecast model

+42 h

Total aerosol mass mixing ratio



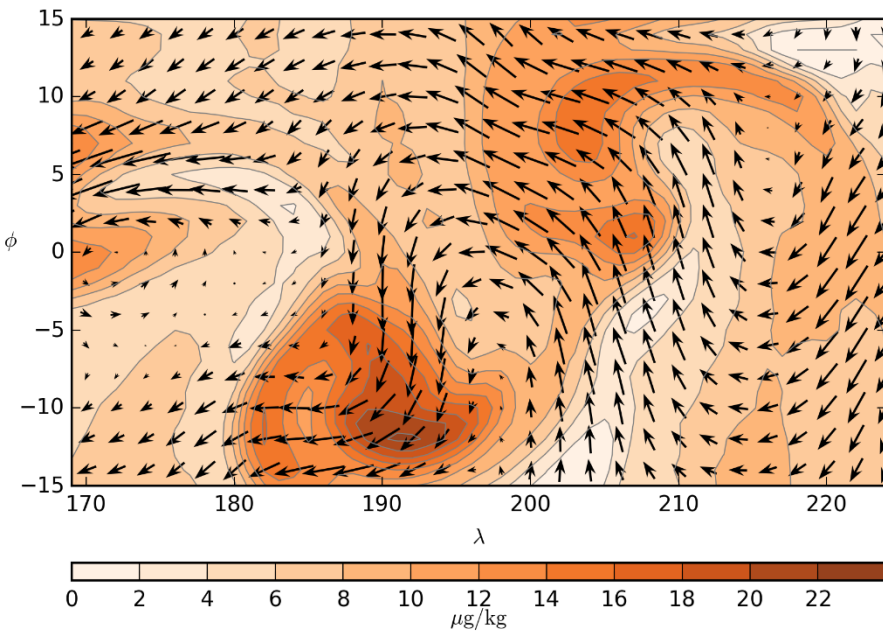
Specific humidity, 6-hour cum. precipitation



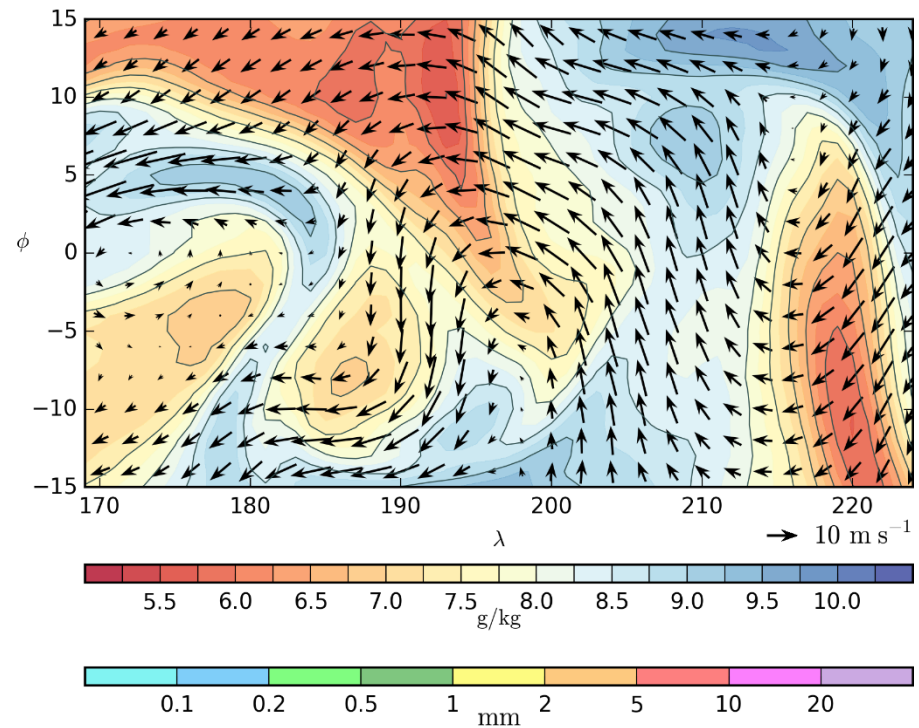
Aerosol-moisture-wind coupling in forecast model

+48 h

Total aerosol mass mixing ratio



Specific humidity, 6-hour cum. precipitation



Assimilation model (MADDAM)

Incremental 4D-Var data assimilation:

$$J(\boldsymbol{\chi}) = \frac{1}{2}(\boldsymbol{\chi} + \boldsymbol{\chi}_g)^T(\boldsymbol{\chi} + \boldsymbol{\chi}_g) + \frac{1}{2} \sum_{k=0}^N (\mathbf{d}_k - \mathbf{H}_k \mathbf{M}_{t_0 \rightarrow t_k} \mathbf{L} \boldsymbol{\chi})^T \mathbf{R}_k^{-1} (\mathbf{d}_k - \mathbf{H}_k \mathbf{M}_{t_0 \rightarrow t_k} \mathbf{L} \boldsymbol{\chi})$$

$\boldsymbol{\chi} = [\boldsymbol{\chi}^d, \boldsymbol{\chi}^q, \boldsymbol{\chi}^c]$ Control vector (three subvectors)

\mathbf{L} , such that $\mathbf{L}^T \mathbf{B}^{-1} \mathbf{L} = \mathbf{I}$ Control variable transform

$\mathbf{d}_k = \mathbf{y}_k - \mathbf{H}_k \mathbf{M}_{t_0 \rightarrow t_k}(\mathbf{x}_g)$ Innovation vector

$\mathbf{x} - \mathbf{x}_g = \delta \mathbf{x} = \mathbf{L} \boldsymbol{\chi}$ Model state increment

$\mathbf{x}_g - \mathbf{x}_b = \delta \mathbf{x}_g = \mathbf{L} \boldsymbol{\chi}_g$ Guess increment

Assimilation model (MADDAM)

Background-error covariance model for dynamics:

- Multivariate balanced analysis of u, v, θ via equatorial linear waves following Žagar et al. (2004, 2008)

$$\begin{bmatrix} \theta(x, y) \\ u(x, y) \\ v(x, y) \end{bmatrix} = \sum_{k=-N_k}^{N_k} \sum_{n=0}^{N_n} \sum_{m=-1}^1 \chi_{knm}^d \begin{bmatrix} \theta_{knm}(y) \\ u_{knm}(y) \\ v_{knm}(y) \end{bmatrix} e^{ikx}$$

- Each horizontal structure function a product of Fourier mode and parabolic cylinder function D_n :

$$v_{knm}(x, y) = D_n \left(\frac{y \sqrt{2}}{L_e} \right) = 2^{-\frac{n}{2}} \exp \left(-\frac{y^2}{2 L_e^2} \right) H_n \left(\frac{y}{L_e} \right)$$

$L_e = \sqrt{c/\beta}$ is equatorial radius of deformation, H_n is Hermite polynomial

- Spectral background-error variance for each mode is estimated from ECMWF EDA (Isaksen, 2010) using Normal modes decomposition software MODES (Žagar, 2015)

Assimilation model (MADDAM)

Background-error covariance model for moisture:

- Control variable = normalized relative humidity (similar to Holm (2002) and Gustafsson (2011))

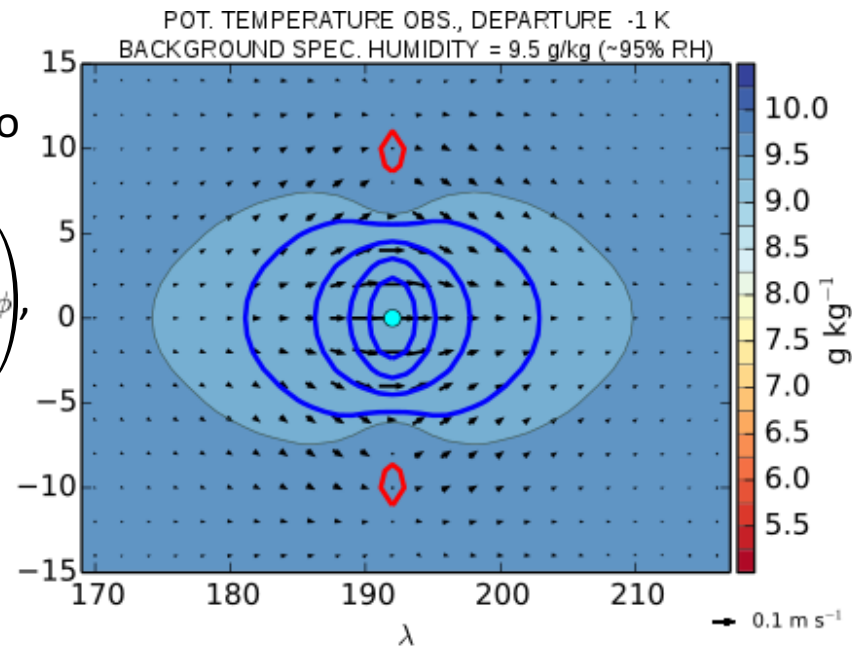
$$\delta RH = -\frac{L_c}{R_v} \frac{RH_g}{T_g^2} \gamma \delta \theta + \frac{1}{q_s(T_g)} \delta q \quad \text{and} \quad \delta RH_n = \frac{\delta RH}{\sigma(RH_b + \delta RH/2)}$$

- Spectral autocorrelation model with anisotropic SOAR corr. function between two distinct point i and j defined as:

$$\rho_{ij} = \left(1 + 2 \sqrt{\frac{\Delta x_{ij}^2}{L_x^2} + \frac{\Delta y_{ij}^2}{L_y^2}} \right) \exp \left(-2 \sqrt{\frac{\Delta x_{ij}^2}{L_x^2} + \frac{\Delta y_{ij}^2}{L_y^2}} \phi \right),$$

$$L_x = 400 \text{ km}, L_y = 200 \text{ km}$$

as zonal flow dominates



Assimilation model (MADDAM)

Background-error covariance model for aerosols:

- Univariate assimilation
- Autocorrelation model the same as for moisture
- Control variable total aerosol mass mixing ratio

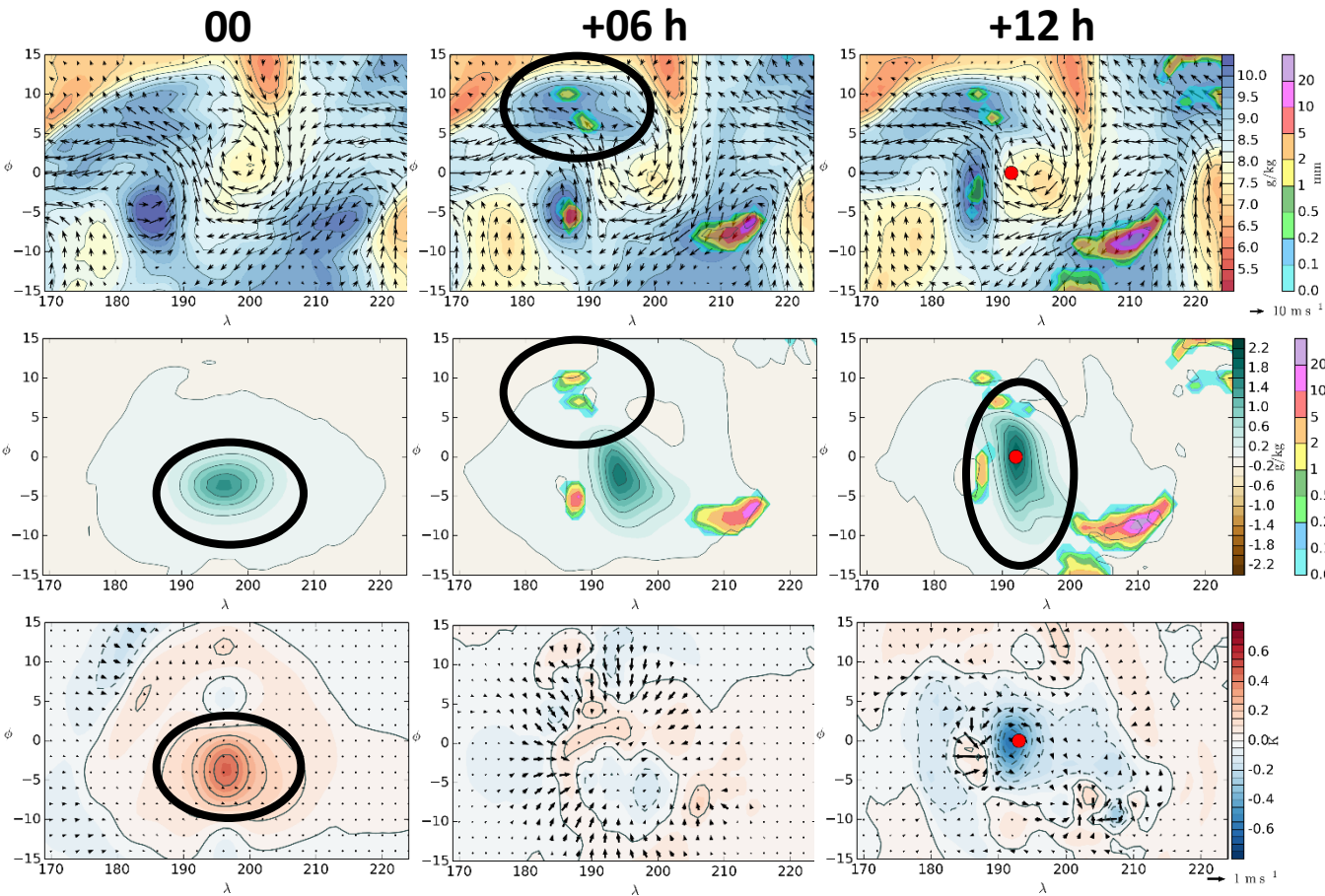
Setup of assimilation experiments

- 12- or 24-hour 4D-Var
- Max. 10 outer loops, max. 20 inner loops
- Observations generated from nature run and are Gaussianly perturbed
- Background and observational errors are similar:

	u [m s^{-1}]	v [m s^{-1}]	θ [K]	RH [%]	c [$\mu\text{g kg}^{-1}$]
σ_b	3.15	2.85	1	5 - 15	$\max(0.2 \bar{c}, 1)$
σ_o	3	3	1	10	$\max(0.2 c, 1)$

Single moisture observation

- Single specific humidity observation (**RED DOT**) at the end of 12-hour 4D-Var window:
 $\Delta q = 2.3 \text{ g kg}^{-1}$



Background specific humidity and 6-hour cumulative precipitation

Specific humidity analysis increment and analysis precipitation

Temperature and wind analysis increments

4D-Var wind tracing: motivation

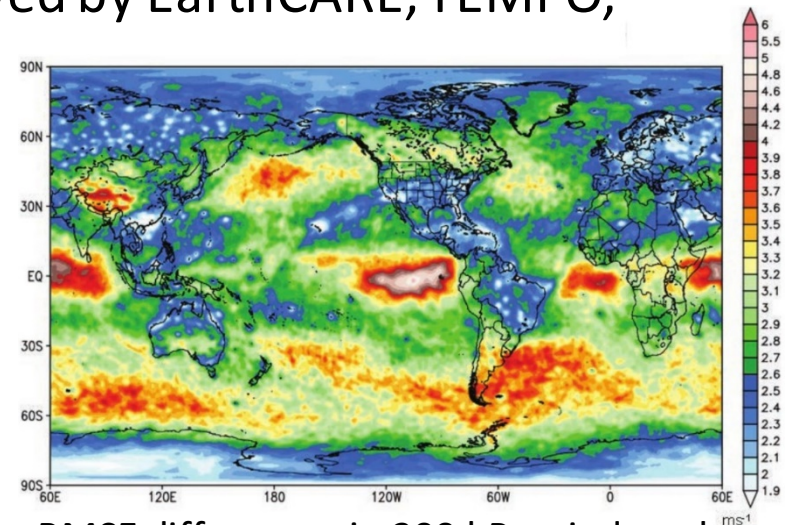
1) An increasing number of atmospheric composition profiles (CALIPSO, Aeolus, Sentinel 5P, followed by EarthCARE, TEMPO, Sentinels 4,5...)

2) NWP has changed lately:

$(\mathbf{v}, p, \rho, T, q)$



Fully-coupled Earth-System prediction system



RMSE differences in 300 hPa wind analyses ECMWF vs GFS (Baker et al., 2014)

3) Need for better wind information in analyses

4) Atmospheric composition and winds dynamically interact

Previous studies on wind tracing

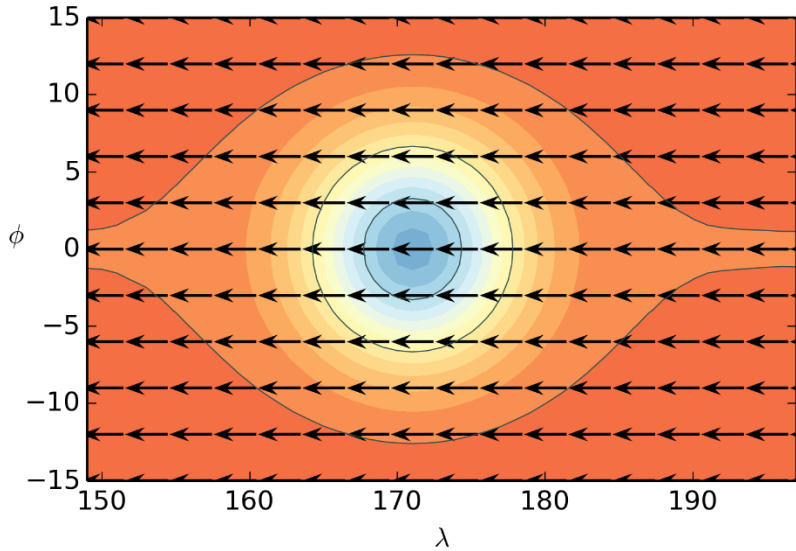
APPROACH	AUTHORS	OUTCOME
Simple 1D and 2D transport models + EKF	Daley (1995, 1996)	First theoretical study of wind tracing. Possible for sufficient variability, high resolution, accurate tracer obs.
Operational 4D-Var + humidity obs.	Andersson et al. (1994), Bormann and Thépaut (2004), Geer et al. (2008), Bonavita and Holm (2016)	Significant wind information retrieved
Idealized & NWP models + simulated perfect obs. of passive tracers	Peuch (2000), Allen (2013, 2014, 2016)	Positive impact on analysis. Analytical 4D-Var solution for tracing (does not take into account accuracy of retrieved tracer spatial gradients)
NWP 4D-Var + ozone obs.	Semane (2009)	Less significant but still positive impact Problem: insufficient availability, accuracy of obs.
Operational 4D-Var + trace gases obs. / aerosol obs.	Benedetti et al. (2009) Han and McNally (2010)	Impact of tracer data on wind sensitivities turned off in assimilation due to model/observation biases

Everywhere at every time!
Wealth of data which are not used to full potential!

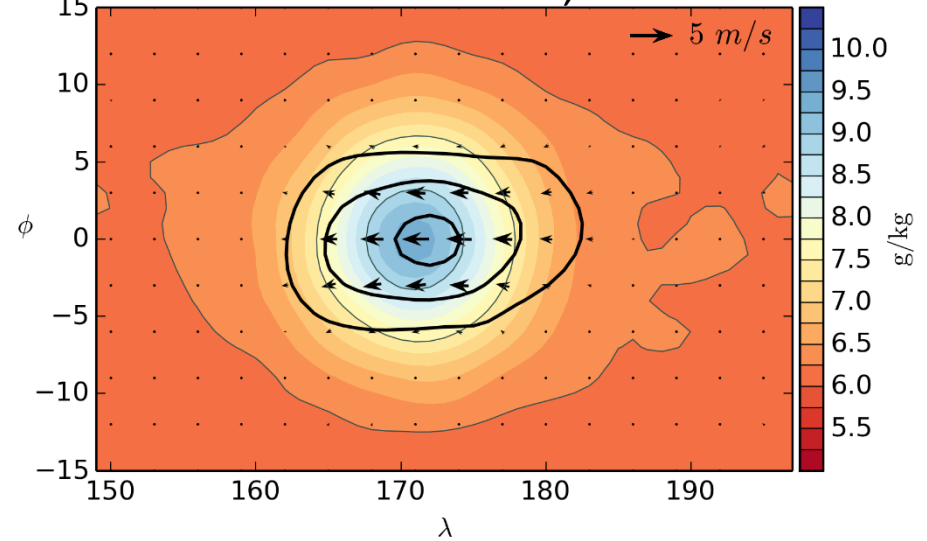


Wind tracing – linear case

TRUTH AT +06 h

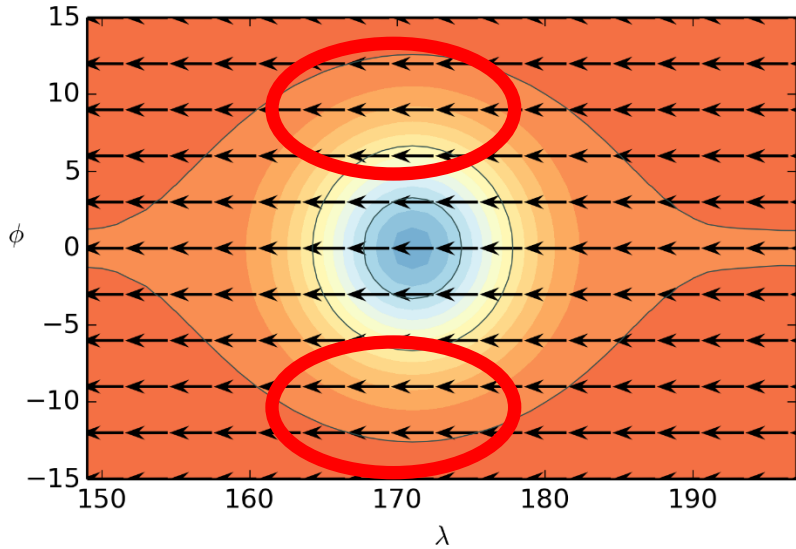


ANALYSIS AT +06 h, 12-HOUR WINDOW
PERT. OBS. 220 km APART, EVERY 1 HOUR

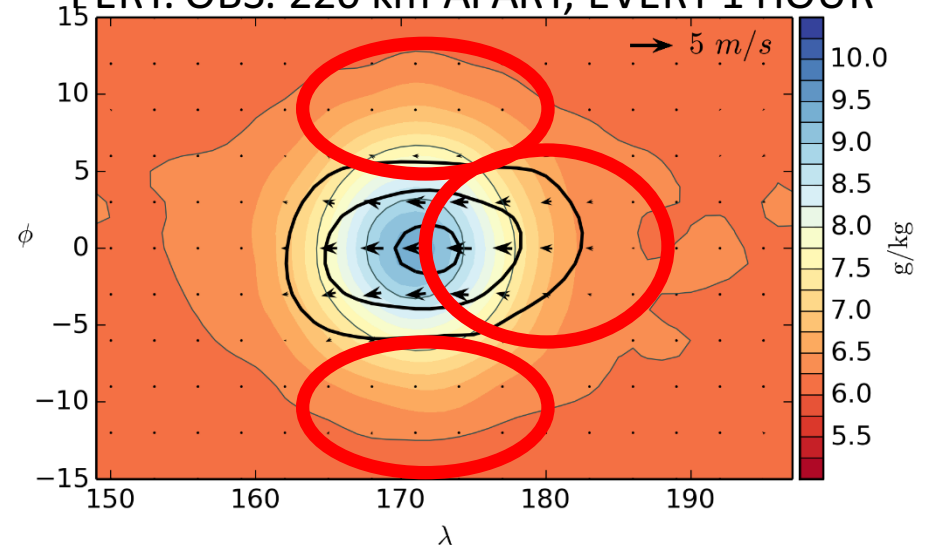


Wind tracing – linear case

TRUTH AT +06 h



ANALYSIS AT +06 h, 12-HOUR WINDOW
PERT. OBS. 220 km APART, EVERY 1 HOUR



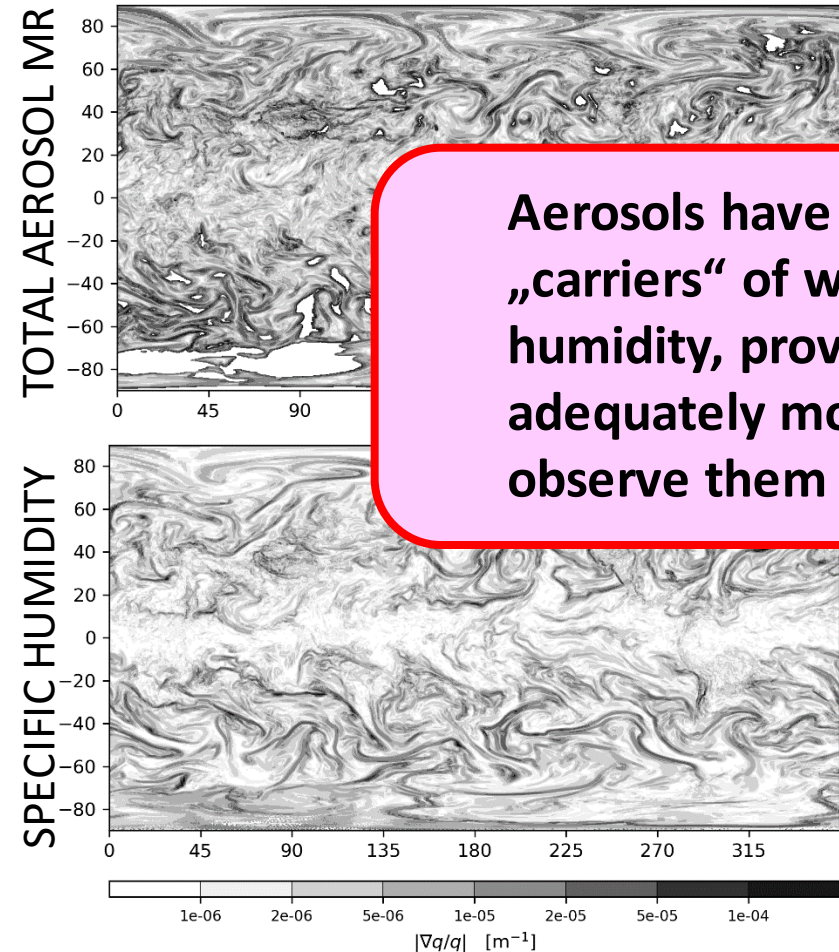
Basic properties of wind tracing:

- 1) $|\delta \mathbf{v}| \propto |q_{obs} - q_g|$
- 2) $|\delta \mathbf{v}| \propto |\nabla q_g| \leftarrow$ determined by humidity observations
- 3) $\mathbf{v} \cdot \nabla q \neq 0$, else no wind is retrieved
- 4) Wind better extracted upwind of the perturbation, where the BKGE covariances are stretched, as the flow accelerates ($\partial u / \partial x > 0$)

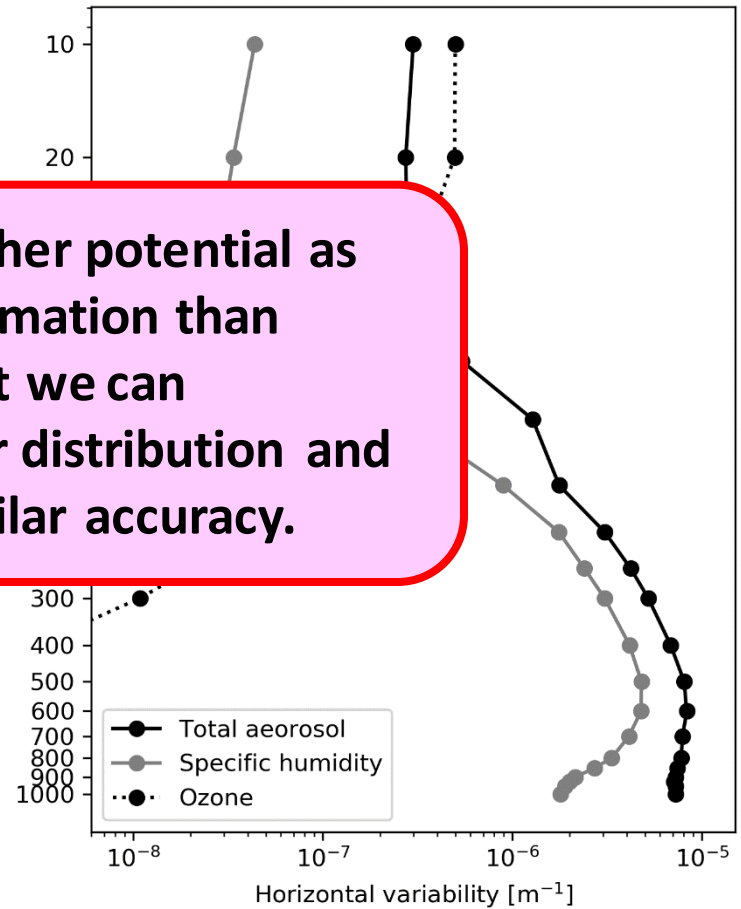
Further motivation

- Aerosols have higher relative spatial gradients $\left| \frac{\nabla c}{c} \right|$ than humidity (data from CAMS)

700 hPa, single time instance



Aerosols have even higher potential as „carriers“ of wind information than humidity, provided that we can adequately model their distribution and observe them with similar accuracy.



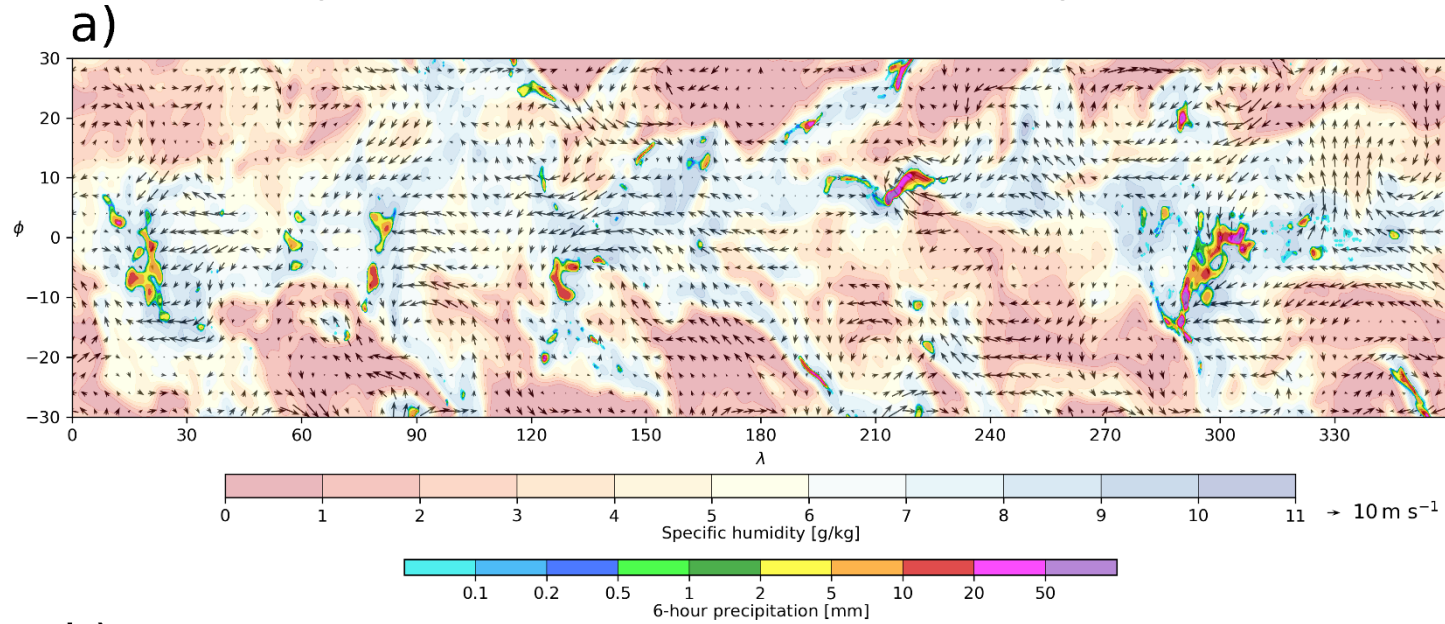
Horizontal axis is logarithmic!!

Ensemble experiments: setup

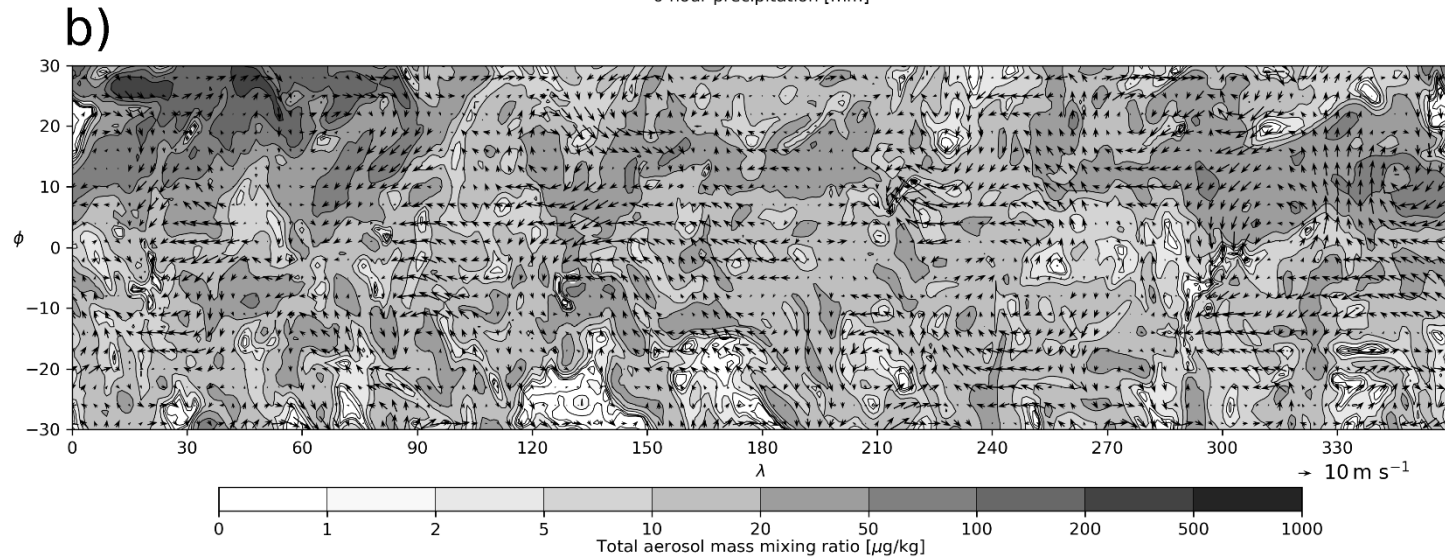
- Fraternal twin OSSE: truth at 0.25° resolution, assimilation at 1°
- 20 ensemble members
- The initial and boundary conditions provided by operational model and then perturbed using the Randomisation method (Andersson, 2000):
 - Potential temperature at 400 hPa
 - Winds at 700 hPa
 - Specific humidity at 700 hPa
 - Total aerosol mass mixing ratio at 700 hPa from CAMS
- Analysis quality measured by
$$\text{NRMSE} = \text{RMSE}(\text{analysis}) / \text{RMSE}(\text{background})$$

Ensemble experiments: setup

Specific humidity and
6-hour cum. prec.

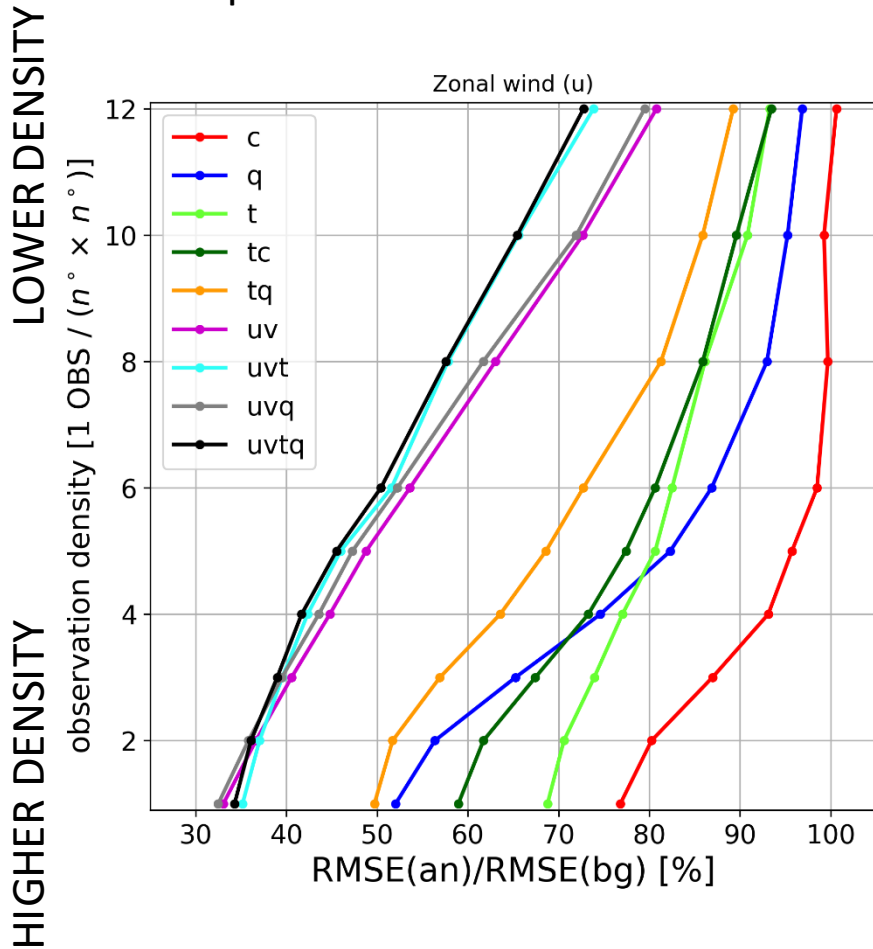


Total aerosol mass
mixing ratio



Impact of the tracer obs. density

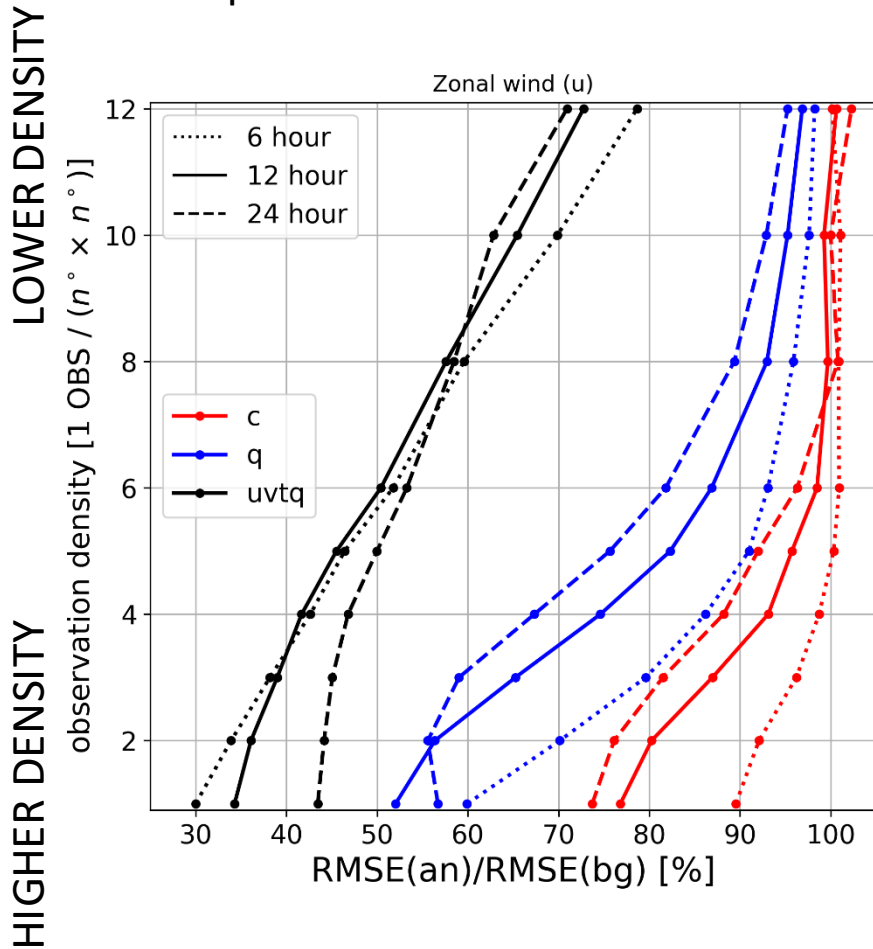
Update interval 1 hour



- 1) More wind extracted from **humidity** obs. than **aerosol** obs. Why?
- 2) Combined **temperature and humidity** observation (**tq**) reduce the wind error by roughly 45-50%, **wind** observations (**uv**) reduce the wind error by 60-65%

Impact of the assim. win. length

Update interval 1 hour



- Wind tracing is better for longer (24-hour) window as the observed tracer distribution changes more
- Information on one observation is spreaded further and is more flow-dependent
- The observed fields are analysed worse for longer windows:
 - model error
 - NRMSE is for longer windows smaller towards the middle of ass. win.

Flow-dependent wind tracing

- **Problem:** feedback of the tracers on wind is switched-off in the TL and AD models in the operational NWP due to model/observational biases. This always deteriorates the tracer analysis [not shown] and neglects the useful wind information in many regions!
- **Idea:** to make better use of existing tracer observations by selectively switching the feedback ON or OFF!
- Locally and temporally manipulate the adjoint and TL models (using function $\theta(x, y, t)$)

$$-\frac{\partial \delta u^*}{\partial t} = +u \frac{\partial \delta u^*}{\partial x} - \delta u^* \frac{\partial u}{\partial x} - \theta(x, y, t) \left(\delta c^* \frac{\partial c}{\partial x} + c \frac{\partial \delta c^*}{\partial x} \right)$$

$$\frac{\partial \delta c}{\partial t} + \theta(x, y, t) \left(\delta u \frac{\partial c}{\partial x} + c \frac{\partial \delta u}{\partial x} \right) + u \frac{\partial \delta c}{\partial x} + \delta c \frac{\partial u}{\partial x} = 0,$$

OBJECTIVE CRITERION:

$$E_{unc} = \frac{1}{N} \sum_{i=1}^N \left| \left(\frac{\partial c}{\partial t} \right)_{phys}^i - \left\langle \frac{\partial c}{\partial t} \right\rangle_{phys} \right|$$

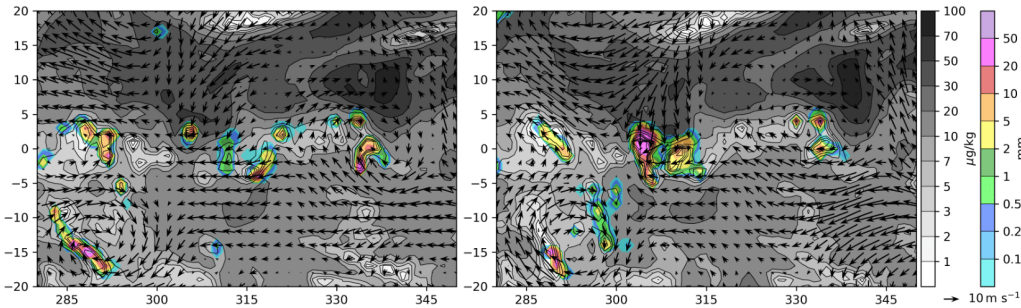
$$\theta = \begin{cases} 0, & E_{unc} > \frac{1}{3} \left| \left\langle \frac{\partial c}{\partial t} \right\rangle_{adv} \right| \\ 1, & \text{otherwise.} \end{cases}$$

Flow-dependent wind tracing

a) +06 h

b) +12 h

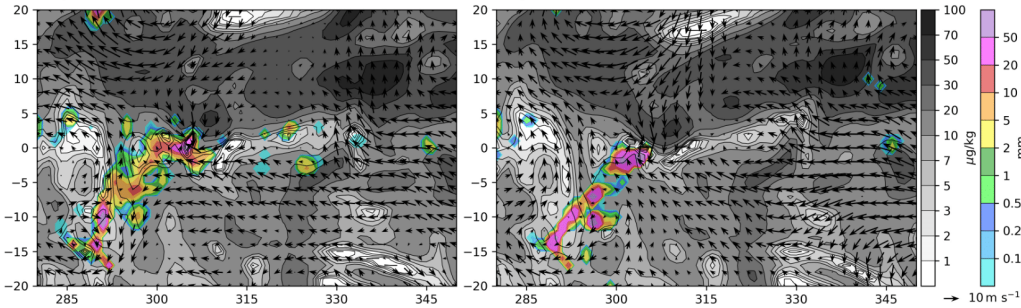
BACKGROUND



c)

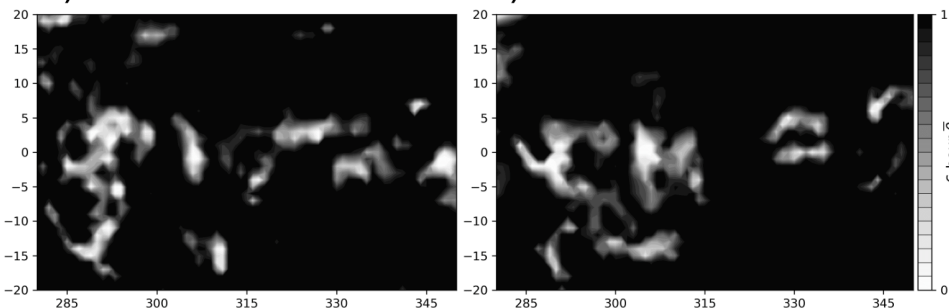
d)

NATURE RUN



e)

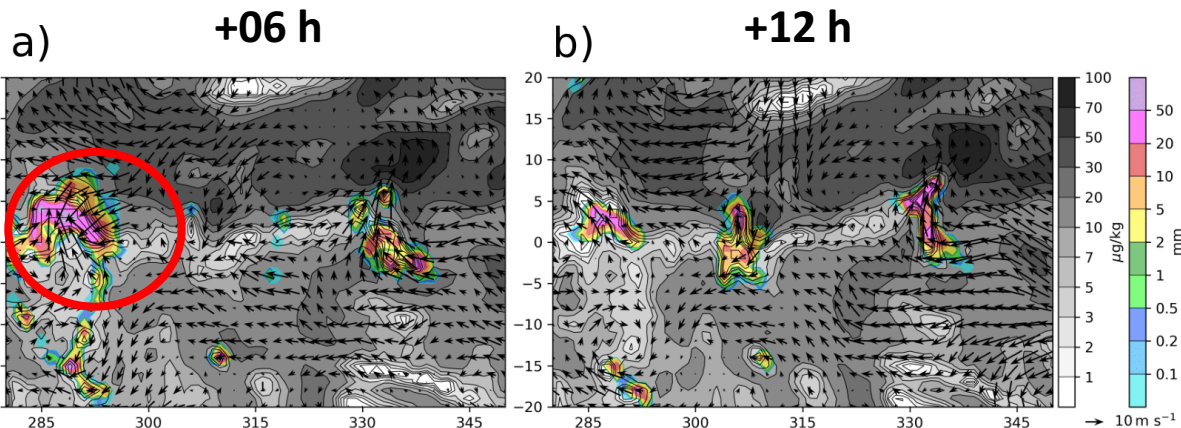
f)



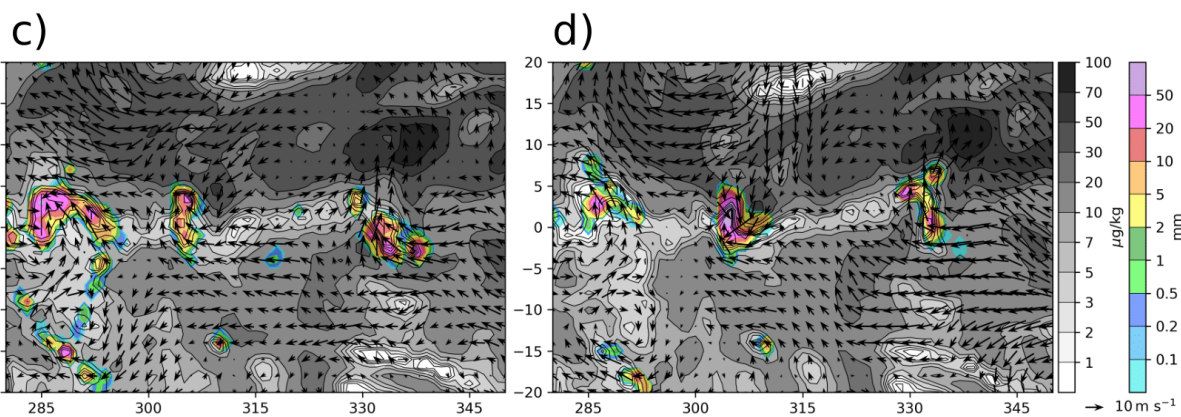
6-hour mean θ

- Background perturbed by randomisation method (50 ens.)
- Wet deposition parameter perturbed (proxy for model error)
- $\theta = 1$ (tracer-wind feedback ON, if physical forcing uncertainty lower than 1/5 of the mean advection rate)

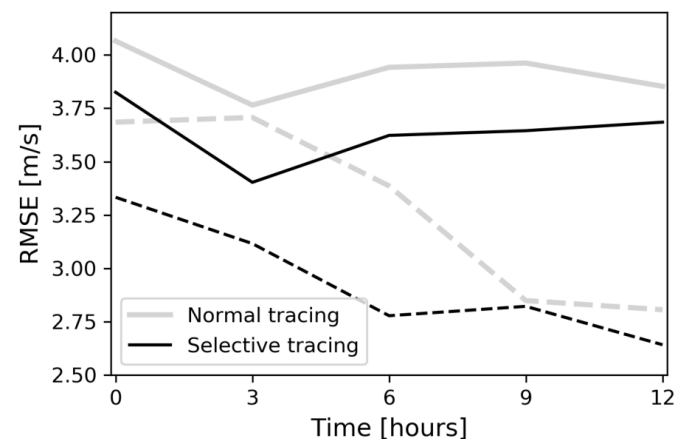
Flow-dependent wind tracing



NORMAL ASSIMILATION
(TRACER-WIND FEEDBACK
SWITCHED ON EVERYWHERE)



FLOW-DEPENDENT (SELECTIVE)
4D-VAR WIND TRACING

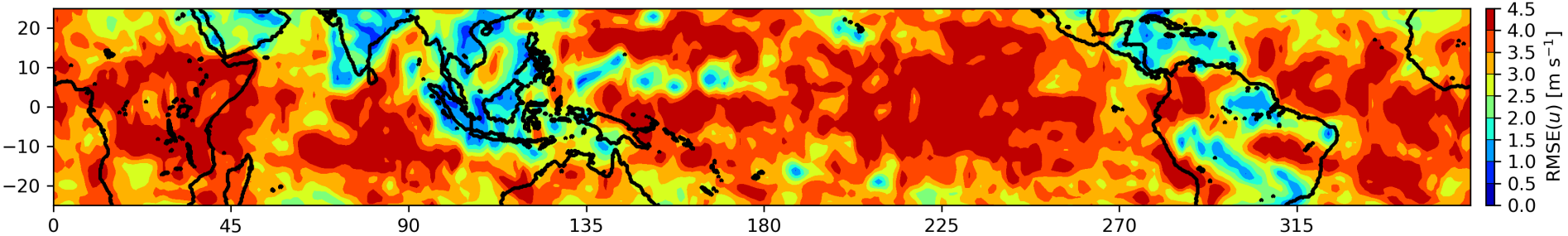


Conclusions

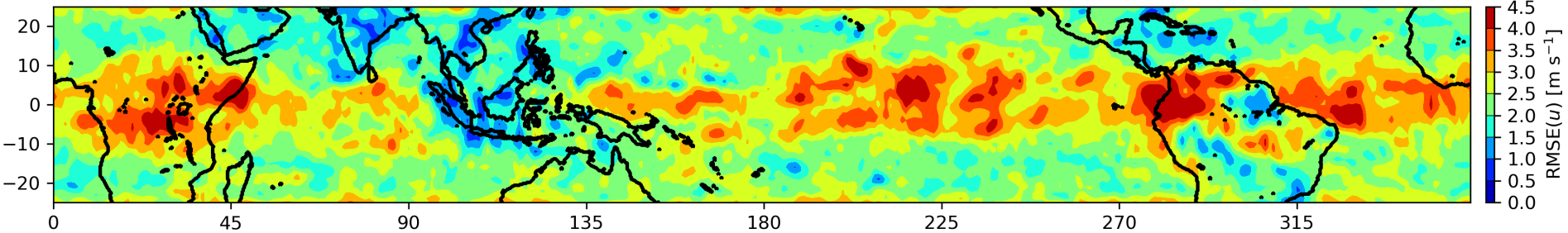
- The MADDAM system has been applied to study 4D-Var wind tracing
- Statistics from ensemble of imperfect model OSSEs:
 - Dense aerosol data alone reduce the analysis wind RMSE by 15-20%, humidity 35-45%, combined temperature-humidity by 45-50%, direct wind observations by 60-65%
 - Longer windows more beneficial, spatial density more important than temporal
 - Significant potential for tracing from aerosol data in dry areas with high advection rate, e.g. tropical and subtropical Atlantic
- **NEW IDEA: Flow-dependent wind tracing shown to improve wind analyses!**
- MADDAM is envisaged to serve as a testbed for new developments in 4D-Var assimilation and a numerical lab for studying the role of inertio-gravity waves and their interaction with humidity across many scales
- Future work: improving the background-error modelling of spatially highly variable fields, e.g. aerosols!
- Intermediate-complexity assimilation systems needed to bridge the gap between assimilation systems with 1D models and NWP!

Zonal wind analysis error for different observation types

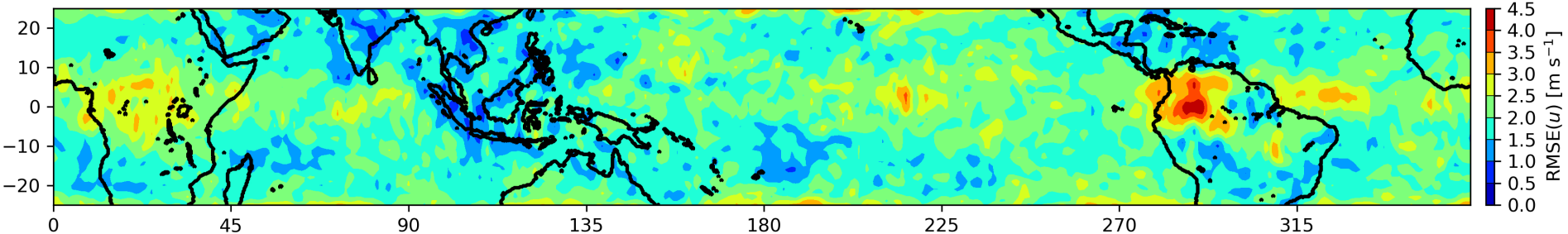
uv from radiosondes (real network used)



uv from radiosondes + dense **T** observations (slow mass-wind adjustment in Tropics)



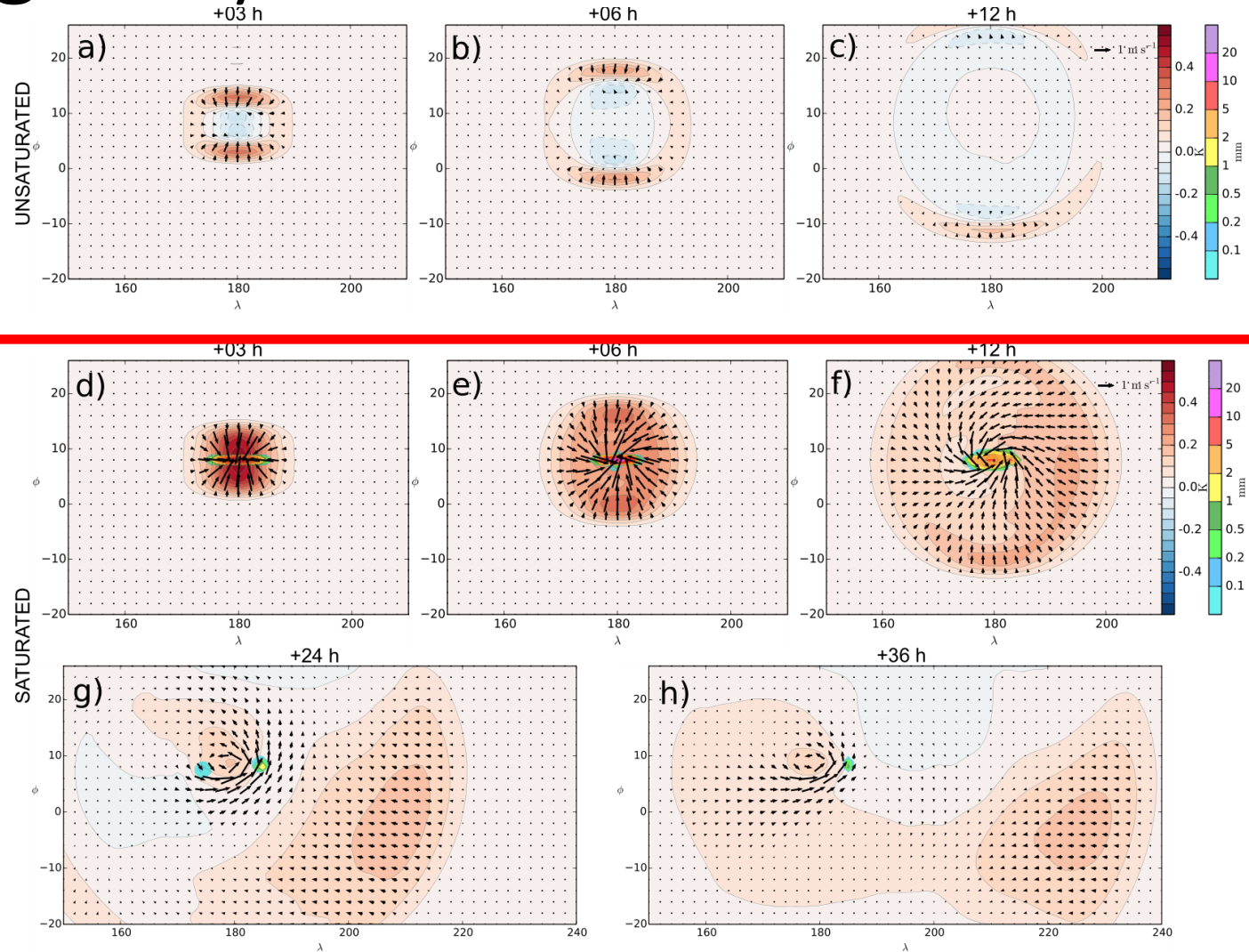
uv from radiosondes + dense **T, q** observations (reduced zonal wind error close to equator)



Thank you!

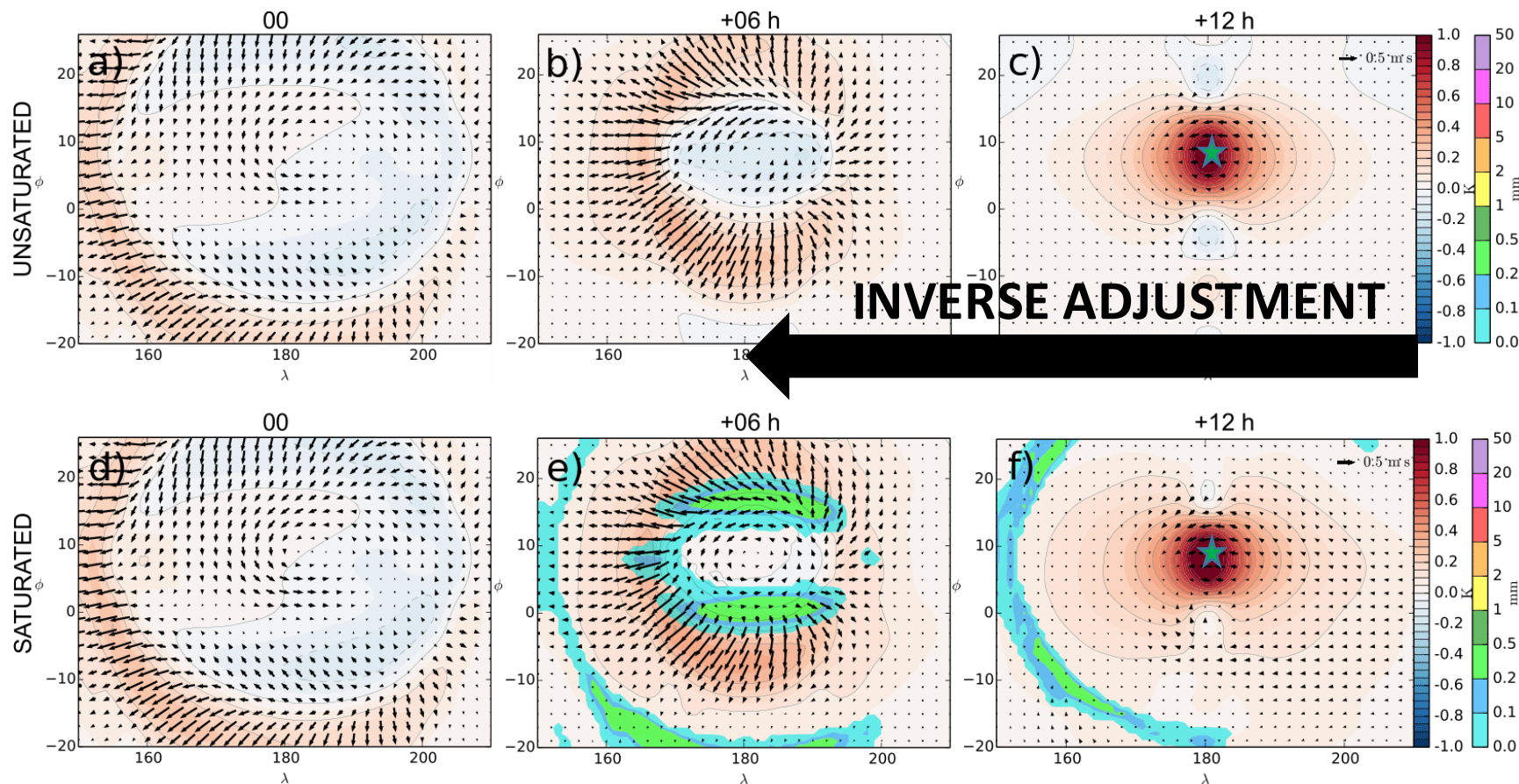
Inertio-gravity waves and 4D-Var

Adjustment to 1 K temperature pert.



Inertio-gravity waves and 4D-Var

- **Assimilation** of temperature perturbation with deperature + 2K at the end of 12-hour time window



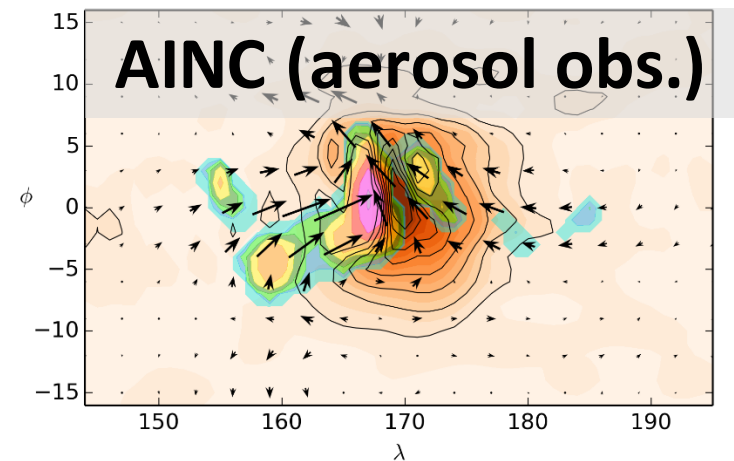
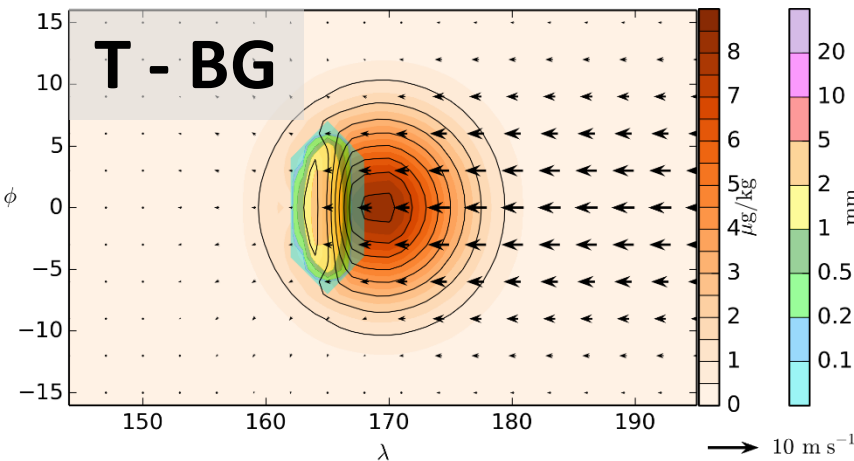
Wind tracing from aerosol obs. in saturated atmosphere

Aerosol, moisture, wind couplings:

- Advection
- Condensation heating
- Wet deposition by precipitation



Despite such simple coupling, 4D-Var can barely resolve the processes due to positive feedback loop occurring



Aerosols and humidity strongly differ as tracers!
Main sink of humidity is directly coupled with winds (and temperature).
Main sink of aerosols is directly coupled to humidity fields (precipitation)



Analysis running into a positive fdb. loop:

- 1) The aerosol conc. is lowered by strong westerly advection
- 2) This leads to convergence and strong precipitation and deposition, which additional lowers the aerosol conc.
- 3) This results in even more convergent flow in next outer loop. → 2)

Wind tracing from aerosol in saturated atmosphere

Aerosol, moisture, wind couplings:

- Advection
- Condensation heating
- Wet deposition



Despite such simple coupling, 4D-Var can barely resolve the processes due to positive feedback between condensation heating and wind tracing

Wind tracing from aerosol data in saturated atmosphere requires:

- Good info on all thermodynamic fields
- Correctly located precipitation areas
- Accurate parametrizations of sink/source processes
- Dense and accurate observations

